Properties of Matter

Prepared by

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توصيف مقرر دراسى

1- بيانات المقرر		
الرمز الكودى :	اسم المقرر: **Properties of Matter**	الفرقة /المستوى :الأولى
التخصص :	عدد الوحدات الدراسية اسبوعيا: نظرى 3 عملى 6	

2- هدف المقرر:	The main objective of this course is to know the basic concepts and techniques of properties of matter which will be necessary for scientific applications.
3- المستهدف من تدريس المقرر :	**Students of Technical Health Institutes**
ا. المعلومات والمفاهيم :	**By the end of this course, graduates must be able to:** • define the basic concepts related to the properties of matter and appreciate how these concepts apply to teeth and bone fields. • describe the advanced solid materials according to their elasticity. • describe the elastic properties of different advanced materials. • state Hook's law of elasticity. • know the advanced applications of physical parameters related to teeth and bone fields. • identify the three fundamental physical quantities: length, mass, and time. • determine the proper units for any physical quantity using dimensional analysis. • know the main differences between the fundamental quantities and the derived quantities. • state the continuity equation for an incompressible fluid. • know the role of properties of matter in other disciplines, including engineering, chemistry, and medicine.
ب- المهارات الذهنية :	**By the end of this course, graduates must be able to:** • synthesize assessment data to formulate teeth and bone diagnoses. • assist patient to make informed health care decisions. • draw a diagram of Hook's law of elasticity. • compare among the types ofelasticity's modulii for advanced materials that are widely used in teeth and bone fields. • synthesize clinical evidence in order to solve problems related to the management of patient care. • use teaching principles in implementing educational activities to patient. • formulate nursing care plan to meet client needs that related to teeth and bone fields. • explain the different types of elasticity's module. • validate any physical equation.
ج- المهارات المهنية الخاصة بالمقرر:	**By the end of this course, graduates must be able to:** • make referrals to appropriate community resources. • measure the outcomes of nursing activities related to teeth and bone fields.

• calculate the different parameters such as stress, strain, shear modulus, bulk modulus, and hardness number that are related to teeth and bone fields. • measure the coefficient of the surface tension of a liquid. • verify Hook's law experimentally for advanced materials. • measure hardness numbers of advanced materials that are widely used teeth and bone fields.	
By the end of this course, graduates must be able to: • develop strong problem-solving skills. • use information technology related to teeth and bone fields. • work effectively with a team. • acquire the ability of self-learning. • apply the communication skills in social and therapeutic context. • apply principles of leadership in different health care settings. • write scientific report about the recent applications of physical parameters related to teeth and bone fields. • acquire the habit of writing down the information given in a problem and those quantities that need to be found. • cooperate with other colleagues and with instructors. • participate other colleagues in the practical activities.	د- المهارات العامة :
Chapter (1): Units and Dimension • Measurement • Units, Standards, and the SI System: **1)** Systems of units **2)** The physical quantities **3)** Conversion of units • Dimensions and dimensional analysis **Chapter (2): Elasticity** • Elastic materials • Inelastic materials • Stress, strain • Hook's Law • Elastic modulus: **1)** Young's modulus **2)** Shear modulus **3)** Bulk modulus • Elasticity and plasticity • Stress-strain graph • Hardness of advanced materials **Chapter (3): Fluid Statics** • Density • Pressure in fluid • Archimedes's principle • Floatation law • Surface tension **Chapter (4): Fluid Dynamic**	4- محتوى المقرر:

• The fluid flow. • Equation of continuity • Bernoulli's equation ☒ special cases of Bernoulli's equation • Viscosity ☒ Poiseuille`s equation ☒ Stock's law • Reynolds number	

• Lectures • Practical experiments (Laboratory work). • Study groups • Discussion group (active learning) • Pair work activities • Making posters (self-learning) • Video demonstration • Data show.	5- أساليب التعليم والتعلم
• Simulation and modeling • Individual guidance • Individual feedback • Remedial programs	6- أساليب التعليم والتعلم للطلاب ذوى القدرات المحدودة
	7- تقويم الطلاب :
a. Class work: 1. Quizzes 2. Midterm theoretical 3. Practical exam 4. Assignments 5. Participation **b.** Final exam: Written theoretical	أ- الأساليب المستخدمة
a. Class work: 1. Quiz I (5th week) 5 marks 2. Attendance 5 marks 3. Midterm theoretical (7th week) 10 marks 4. Clinical work: 30 marks b. Final exam Practical exam (13th week) 10 marks written theoretical exam (15th week) 90 marks	ب- التوقيت
Case records and reports (5 marks) Quiz : 5 mark Midterm: 10 marks Attendance 5 marks Clinical: 20 marks Clinical exam:15 marks Final written exam 90 marks. Total percentage 150 mark	ج- توزيع الدرجات
	8- قائمة الكتب الدراسية والمراجع :
	أ- مذكرات

ب- كتب ملزمة		
ج- كتب مقترحة	1. R. A. Serway and J. W. Jewett, "Physics for Scientists and Engineers", 8th Edition, Thomson Brooks/Cole., (2008) 2. F. W. Sears, M. W. Zemansky and H. D. Young, "College Physics", 7th Edition, Addison-Wesley Publishing Company, Inc., (1991) 3. Fundamentals of physics, David Halliday, Robert Resnick, Jearl Walker., JohnWiley & Sons, Inc., 8th Ed., 2008.	
د- دوريات علمية أو نشرات ...... الخ		

Contents

Chapter (1)
Units and Dimension

Like all other sciences, physics is based on experimental observations and quantitative measurements. The main objectives of physics are to identify a limited number of fundamental laws that govern natural phenomena and use them to develop theories that can predict the results of future experiments. The fundamental laws used in developing theories are expressed in the language of mathematics, the tool that provides a bridge between theory and experiment.

1.1 Measurement

To describe natural phenomena, we must make measurements of various aspects of nature. Each measurement is associated with a physical quantity, such as the length of an object. The laws of physics are expressed as mathematical relationships among physical quantities that we will introduce and discuss throughout this chapter.

1.2 Units, Standards, and the SI System

The Physical Quantities

In mechanics, the three fundamental quantities are length, mass, and time. All other quantities in mechanics can be expressed in terms of these three.

If we are to report the results of a measurement to someone who wishes to reproduce this measurement, a ***standard*** must be defined. It would be meaningless if a visitor from another planet were to talk to us about a length of 8 “glitches” if we do not know the meaning of the unit glitch. On the other hand, if someone familiar with our system of measurement reports that a wall is 2 meters high and our unit of length is defined to be 1 meter, we know that the height of the wall is twice our basic length unit. Measurement standards used by different people in different places—throughout the Universe—must yield the same result. In addition, standards used for measurements must not change with time.

The variables length, time, and mass are examples of ***fundamental quantities***. Most other variables are ***derived quantities***, those that can be expressed as a mathematical combination of fundamental quantities. Common examples are *area* (a product of two lengths) and *speed* (a ratio of a length to a time interval).

Systems of Units

Several systems of units have been in use over the years. Today the most important by far is the **Système International** (French for International System). It is abbreviated **SI**, and its fundamental units of length, mass, and time are the ***meter***, ***kilogram***, and ***second***, respectively. This system used to be called the **MKS** (meter-kilogram-second) system. Other standards for SI fundamental units established by the committee are those for temperature (the ***kelvin***), electric current (the ***ampere***), luminous intensity (the ***candela***), and the amount of substance (the ***mole***).

A second metric system is the **cgs system**, in which the centimeter, gram, and second are the standard units of length, mass, and time, as abbreviated in the title.

Another system of units, the *U.S. customary system*, is still used in the United States despite acceptance of SI by the rest of the world. In this system, the units of length, mass, and time are the ***foot*** (ft), ***slug***, and ***second***, respectively.

SI units are the principal ones used today in scientific work and industry. We will therefore use SI units almost exclusively in this course.

In addition to the fundamental SI units of meter, kilogram, and second, we can also use other units, such as millimeters and nanoseconds, where the prefixes *milli*- and *nano*- denote multipliers of the basic units based on various powers of ten. Prefixes for the various powers of ten and their abbreviations are listed in Table 1.1. For example, 10^{-3} m is equivalent to 1 millimeter (mm), and 10^{3} m corresponds to 1 kilometer (km). Likewise, 1 kilogram (kg) is 10^{3} grams (g), and 1 mega volt (MV) is 10^{6} volts (V).

Table 1.1 Prefixes for Powers of Ten

Power	Prefix	Abbreviation	Power	Prefix	Abbreviation
10^{-24}	yocto	y	10^{3}	kilo	k
10^{-21}	zepto	z	10^{6}	mega	M
10^{-18}	atto	a	10^{9}	giga	G
10^{-15}	femto	f	10^{12}	tera	T
10^{-12}	pico	p	10^{15}	peta	P
10^{-9}	nano	n	10^{18}	exa	E
10^{-6}	micro	μ	10^{21}	zetta	Z
10^{-3}	milli	m	10^{24}	yotta	Y
10^{-2}	centi	c			
10^{-1}	deci	d			

1.3 Conversion of Units

Sometimes it is necessary to convert units from one measurement system to another or convert within a system (for example, from kilometers to meters). Conversion factors between SI and U.S. customary units of length are as follows:

$$1 \text{ mile} = 1\,609 \text{ m} = 1.609 \text{ km} \qquad 1 \text{ ft} = 0.304\,8 \text{ m} = 30.48 \text{ cm}$$
$$1 \text{ m} = 39.37 \text{ in.} = 3.281 \text{ ft} \qquad 1 \text{ in.} = 0.025\,4 \text{ m} = 2.54 \text{ cm (exactly)}$$

Units can be treated as algebraic quantities that can cancel each other. For example, suppose we wish to convert 15 in. to centimeters. Because 1 in. is defined as exactly 2.54 cm, we find that

$$15.0 \text{ in.} = (15.0 \cancel{\text{in.}})\left(\frac{2.54 \text{ cm}}{1 \cancel{\text{in.}}}\right) = 38.1 \text{ cm}$$

where the ratio in parentheses is equal to 1. We express 1 as 2.54 cm/1 in. (rather than 1 in./2.54 cm) so that the unit "inch" in the denominator cancels with the unit in the original quantity. The remaining unit is the centimeter, our desired result.

1.4 Dimensions and Dimensional Analysis

In physics, the word ***dimension*** denotes the physical nature of a quantity. The distance between two points, for example, can be measured in feet, meters, or furlongs, which are all different ways of expressing the dimension of length.

The symbols we use in this text to specify the dimensions of length, mass, and time are L, M, and T, respectively. We shall often use brackets [ ] to denote the dimensions of a physical quantity. For example, the symbol we use for speed in this book is *v*, and in our notation, the dimensions of speed are written [*v*] = L/T. As another example, the dimensions of area *A* are [*A*] = L^2. The dimensions and units of area, volume, speed, and acceleration are listed in Table 1.2. The dimensions of other quantities, such as force and energy, will be described as they are introduced in the text.

Table 1.2 Dimensions and Units of Four Derived Quantities

Quantity	Area (A)	Volume (V)	Speed (v)	Acceleration (a)
Dimensions	L^2	L^3	L/T	L/T^2
SI units	m^2	m^3	m/s	m/s^2
U.S. customary units	ft^2	ft^3	ft/s	ft/s^2

In many situations, you may have to check a specific equation to see if it matches your expectations. A useful procedure for doing that, called **dimensional analysis**, can be used because dimensions can be treated as algebraic quantities. For example, quantities can be added or subtracted only if they have the same dimensions. Furthermore, the terms on both sides of an equation must have the same dimensions. By following these simple rules, you can use dimensional analysis to determine whether an expression has the correct form. Any relationship can be correct only if the dimensions on both sides of the equation are the same.

To illustrate this procedure, suppose you are interested in an eq uation for the position *x* of a car at a time t if the car starts from rest at *x* = 0 and moves with constant acceleration *a*. The correct expression for this situation is $x = \frac{1}{2}at^2$. The quantity *x* on the left side has the dimension of length. For the equation to be dimensionally correct, the quantity on the right side must also have the dimension of length. We can perform a dimensional check by substituting the dimensions for acceleration, L/T^2 (Table 1.2), and time, T, into the equation. That is, the dimensional form of the equation $x = \frac{1}{2}at^2$ is

$$L = \frac{L}{\cancel{T^2}} \cdot \cancel{T^2} = L$$

The dimensions of time cancel as shown, leaving the dimension of length on the right-hand side to match that on the left.

A more general procedure using dimensional analysis is to set up an expression of the form

$$x \propto a^n t^m$$

where n and m are exponents that must be determined and the symbol α indicates a proportionality. This relationship is correct only if the dimensions of both sides are the same. Because the dimension of the left side is length, the dimension of the right side must also be length. That is,

$$[a^n t^m] = L = L^1T^0$$

Because the dimensions of acceleration are L/T^2 and the dimension of time is T, we have

$$(L/T^2)^n T^m = L^1T^0 \quad \rightarrow \quad (L^n T^{m-2n}) = L^1T^0$$

The exponents of L and T must be the same on both sides of the equation. From the exponents of L, we see immediately that $n = 1$. From the exponents of T, we see that $m - 2n = 0$, which, once we substitute for n, gives us $m = 2$. Returning to our original expression $x \propto a^n t^m$, we conclude that $x \propto at^2$.

Quick Quiz 1.2

True or False: Dimensional analysis can give you the numerical value of constants of proportionality that may appear in an algebraic expression.

Example 1.1 Analysis of an Equation

Show that the expression $v = at$, where v represents speed, a acceleration, and t an instant of time, is dimensionally correct.

Solution

Identify the dimensions of v from Table 1.3:

$$[v] = \frac{L}{T}$$

Identify the dimensions of a from Table 1.3 and multiply by the dimensions of t :

$$[at] = \frac{L}{T^{\cancel{2}}}\cancel{T} = \frac{L}{T}$$

Therefore, $v = at$ is dimensionally correct because we have the same dimensions on both sides. (If the expression were given as $v = at^2$, it would be dimensionally incorrect. Try it and see!)

Example 1.2 Analysis of a Power Law

Suppose we are told that the acceleration a of a particle moving with uniform speed v in a circle of radius r is proportional to some power of r, say r^n, and some power of v, say v^m. **Determine** the values of n and m and **write** the simplest form of an equation for the acceleration.

Solution

Write an expression for a with a dimensionless constant of proportionality k:

$$a = kr^n v^m$$

Substitute the dimensions of a, r, and v:

$$\frac{L}{T^2} = L^n\left(\frac{L}{T}\right)^m = \frac{L^{n+m}}{T^m}$$

Equate the exponents of L and T so that the dimensional equation is balanced:

$$n + m = 1 \text{ and } m = 2$$

Solve the two equations for n:

$$n = -1$$

Write the acceleration expression:

$$a = kr^{-1}v^2 = k\frac{v^2}{r}$$

Problems

1. The position of a particle moving under uniform acceleration is some function of time and the acceleration. Suppose we write this position $s = ka^m t^n$, where k is a dimensionless constant. **Show** by dimensional analysis that this expression is satisfied if $m = 1$ and $n = 2$. Can this analysis give the value of k?

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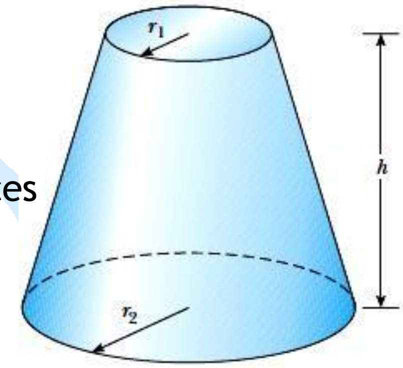
2. The opposite Figure shows a frustrum of a cone.



Of the following geometrical expressions, **which** describes

(a) the total circumference of the flat circular faces

(b) the volume (c) the area of the curved surface?

(i) $\pi(r_1 + r_2)[h^2 + (r_1 - r_2)^2]^{1/2}$

(ii) $2\pi(r_1 + r_2)$

(iii) $\pi h(r_1{}^2 + r_1 r_2 + r_2{}^2)$.

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3. **Which** of the following equations are dimensionally correct?

(a) $v_f = v_i + ax$

(b) $y = (2\text{ m})\cos(kx)$, where $k = 2\text{ m}^{-1}$.

---

4. Newton's law of universal gravitation is represented by

$$F = \frac{GMm}{r^2}$$

Here F is the magnitude of the gravitational force exerted by one small object on another, M and m are the masses of the objects, and r is a distance. Force has the SI units kg.m/s^2. **What** are the SI units of the proportionality constant G?

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5. A worker is to paint the walls of a square room 8 ft high and 12 ft along each side. **What** surface area in square meters must she cover?

---

6. **Calculate** the mass of a solid copper sphere that has a diameter of 3 cm. The density of copper is 8920 kg/m^3

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7. **What** is the mass of a solid iron wrecking ball of radius 18 cm?. The density of iron is 7860 kg/m^3

8. **What** is the mass of a solid iron wrecking ball of radius 18 cm?. The density of iron is 7860 kg/m^3

9. **Express** the following using the prefixes of Table 1.1:
(a) 10^6 volts, (b) 10^{-6} meters, (c) 5×10^2 days, and (d) 3×10^{-9} pieces.

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10. **Write** the following as full (decimal) numbers with standard units:
(a) 35 mm, (b) 25 μV, (c) 250 mg, and (d) 500 picoseconds;
(e) 2.5 femtometers; and (f) 25 gigavolts.

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11. (a) **How many** seconds are there in 1 year? (b) **How many** nanoseconds are there in 1 year? (c) **How many** years are there in 1 second?

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12. (a) **How many** centimeters are there in one kilometers? **How many** millimeters in a kilometer?

Chapter (2) Elasticity

Elastic Materials

In physics, **elasticity** is the ability of a material to resist a distorting influence and to return to its original size and shape when that influence or force is removed. Solid objects will deform when adequate forces are applied on them. If the material is elastic, the object will return to its initial shape and size when these forces are removed.

Most materials which possess elasticity in practice remain purely elastic only up to very small deformations. Therefore, elastic materials are materials that return to its original shape and size after distorting force has been removed or after deformation.

2.1 Inelastic Materials




They are materials that do not return to its original shape and size after distorting force has been removed or after deformation.



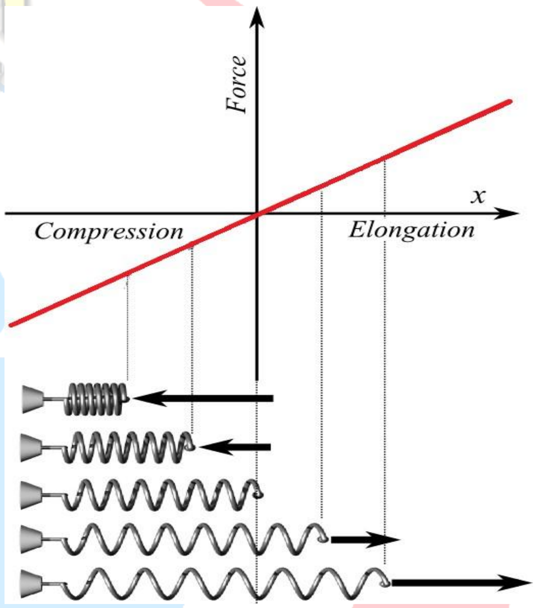
2.2 Hooke's Law

This law is named after 17th-century British physicist **Robert Hooke (1635- 1703)**. He first stated the law in 1676 as a Latin anagram and published the solution of his anagram in 1678 as "the extension is proportional to the force".

Hooke's law states that the force F needed to extend or compress a spring by some distance x scales linearly with respect to that distance. That is:

$$F_s = -kx \tag{2.1}$$

where x is the position of the block relative to its equilibrium (x = 0) position and k is a positive constant called the **force constant** or the **spring constant** of the spring. In other words, the force required to stretch or compress a spring is proportional to the amount of stretch or compression x. This force law for springs is known as **Hooke's law**. The value of k is a measure of the stiffness of the spring. Stiff springs have large k values, and soft springs have small k values. As can be seen from Equation (2.1), the units of k are N/m. Because the spring force always acts toward the equilibrium position (x = 0), it is sometimes called a ***restoring force***.



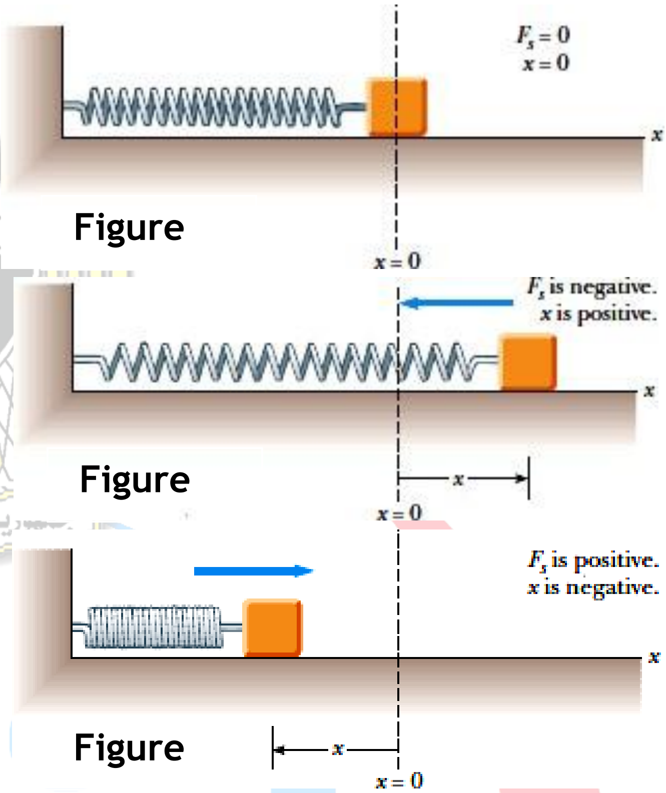
Equation (2.1) holds (to some extent) in many other situations where an elastic body is deformed, such as a musician plucking a string of a guitar and the filling of a party balloon. An elastic material for which this equation can be assumed is said to be linear-elastic or Hookean.

The negative sign in Equation (2.1) signifies that the force exerted by the spring is always directed *opposite* to the displacement from equilibrium.

When $x = 0$ as in Figure (A), the spring is unstretched and $F_s = 0$.

When $x > 0$ as in Figure (B), so that the block is to the right of the equilibrium position, the spring force is directed to the left, in the negative x direction.

When $x < 0$ as in Figure (C), the block is to the left of equilibrium and the spring force is directed to the right, in the positive x direction.

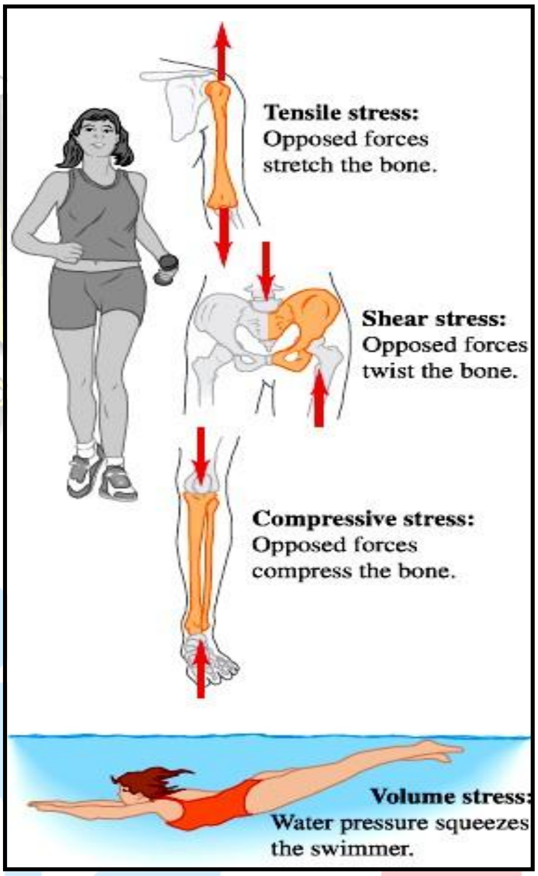


On the other hand, Hooke's law is an accurate approximation for most solid bodies, as long as the forces and deformations are small enough. For this reason, Hooke's law is extensively used in all branches of science and engineering, and is the foundation of many disciplines such as seismology and acoustics. It is also the fundamental principle behind the spring scale, the manometer and the balance wheel of the mechanical clock.

2.3 Elastic Properties of Materials

The rigid body is a useful idealized model, but the stretching, squeezing, and twisting of real objects when forces are applied are often too important to ignore.

Examples of these types of forces are shown in the opposite Figure. We will discuss the deformation of solids in terms of the concepts of stress and strain. **Stress** is a quantity that is proportional to the force causing a deformation. More specifically, stress σ is the external force F acting on an object per unit cross sectional area A:



$$\sigma = \frac{F}{A} \qquad (2.2)$$

Thus, the stress is said to be directly proportional to the load and inversely proportional to the cross-sectional area. The SI unit of stress is the newton per square meter (N/m^2). The units of stress are the same as those of pressure, which we will encounter often in the next chapter.

The usefulness of the concept of stress is apparent. It is not sufficient merely to state the load or force that is being applied to a dental material, because the stress that is produced in the material depends just as much on the cross-sectional area on which the load is acting as it does on the load itself.

The result of a stress is **strain**, which is a measure of the degree of deformation. Strain is dimensionless (no units). It is found that, for sufficiently small stresses, **stress is proportional to strain**; the constant of proportionality depends on the material being deformed and on the nature of the deformation. We call this proportionality constant the **elastic modulus**. We shall discuss this parameter in the following section.

2.4 Elastic Moduli:

The elastic modulus is therefore defined as the ratio of the stress to the resulting strain:

$$\text{Elastic modulus} = \frac{\text{stress}}{\text{strain}} \qquad (2.3)$$

The elastic modulus in general relates what is done to a solid object (a force is applied) to how that object responds (it deforms to some extent).

Elastic modulus is very important parameter when selecting materials in dentistry. In order to ensure the positive therapeutic effect, dental material must possess an elastic modulus value equal or similar to the modulus of dentin (15 - 25 GPa) or enamel (83 GPa) depending on application.

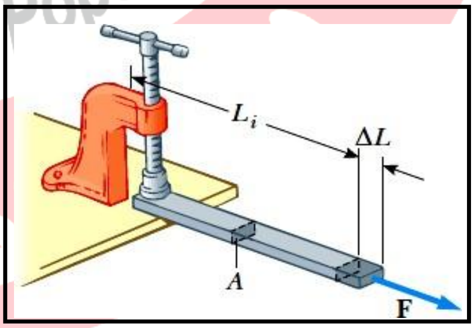
Higher value of the elastic modulus indicates a higher stiffness and lower flexibility of the cast construction of partial denture or other prostheses.

We consider three types of deformation and define an elastic modulus for each:

- **Young's modulus** measures the resistance of a solid to a change in its length.
- **Shear modulus** measures the resistance to motion of the planes within a solid parallel to each other.
- **Bulk modulus** measures the resistance of solids or liquids to changes in their volume.

2.4.1 Young's Modulus: Elasticity inLength

Consider a long bar of cross-sectional area *A* and initial length L_i that is clamped at one end as in the opposite Figure.



When an external force is applied perpendicular to the cross section, internal molecular forces in the bar resist distortion ("stretching"), but the bar reaches an equilibrium situation in which its final length L_f is greater than L_i and in which the external force is exactly balanced by the internal forces. In such a situation, the bar is said to be stressed.

We define the **tensile stress** as the ratio of the magnitude of the external force F to the cross-sectional area A, where the cross section is perpendicular to the force vector. The **tensile strain** in this case is defined as the ratio of the change in length ΔL to the original length L_i. We define **Young's modulus** by a combination of these two ratios:

$$Y = \frac{\text{tensile stress}}{\text{tensile strain}} = \frac{F/A}{\Delta L/L_i} \qquad (2.4)$$

Young's modulus is typically used to characterize a rod or wire stressed under either tension or compression. Because strain is a dimensionless quantity, Y has units of force per unit area.

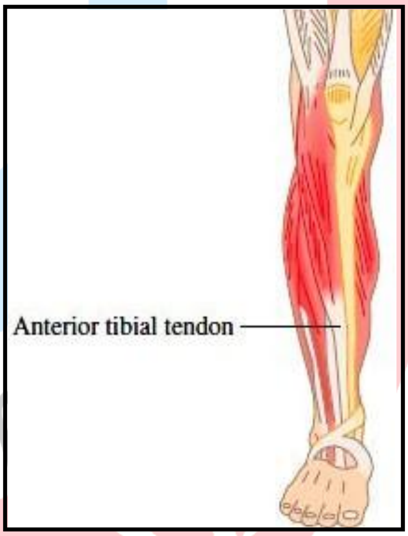
Quick Quiz 2.1

While lifting a load, the steel cable of a crane stretches by 1 cm. if you want the cable to stretch by only 0.5 cm, by what factor must you increase its diameter?

(a) $\sqrt{2}$ (b) 2 (c) $2\sqrt{2}$ (d) 4

Application: Young's Modulus of a Tendon

The anterior tibial tendon connects your foot to the large muscle that runs along the side of your shinbone. (You can feel this tendon at the front of your ankle.) Measurements show that this tendon has a Young's modulus of 1.2×10^9 Pa. Hence this tendon stretches substantially (up to 2.5% of its length) in response to the stresses experienced in walking and running.



Example 2.1 Tensile stress and strain

A steel rod 2 m long has a cross-sectional area of .0.3 cm^2 It is hung by one end from a support, and a 550-kg milling machine is hung from its other end. Determine the stress on the rod and the resulting strain and elongation. Young's modulus of steel is 2×10^{11} N/m^2

Solution

The rod is under tension, so we can use the following Equation to find the tensile stress:

We can use Equation 2.4 to find the corresponding tensile strain:

$$\text{Tensile stress} = \frac{F}{A} = \frac{(550\ kg)(9.8\ m/s^2)}{0.3\times10^{-4}m^2} = 1.8\times10^{8}\ N/m^2$$

$$\text{Tensile strain} = \frac{\text{Tensile stress}}{Y} = \frac{1.8\times10^{8}\ N/m^2}{2\times10^{11}\ N/m^2} = 9\times10^{-4}$$

Use the following Equation to find the resulting elongation:

$$\text{Tensile strain} = \frac{\Delta L}{L_i}$$

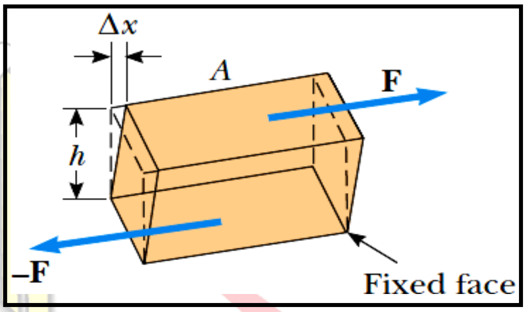
$$\therefore \Delta L(\text{Elongation}) = (\text{Tensile stain})\times L_i$$

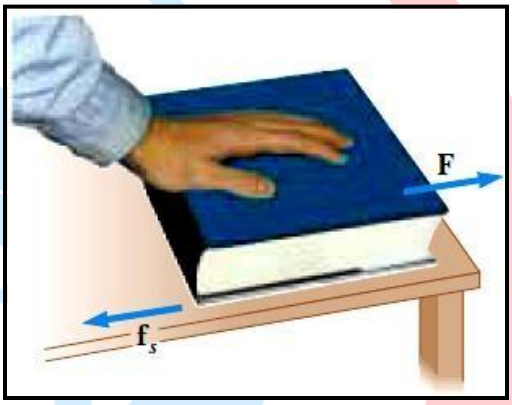
$$\therefore \Delta L = (9\times10^{-4})(2\ m) = 18\times10^{-4}\ m = 1.8\ mm$$

This small elongation, resulting from a load of over half a ton, is a testament to the stiffness of steel.

2.4.2 Shear Modulus: Elasticity of Shape

Another type of deformation occurs when an object is subjected to a force parallel to one of its faces while the opposite face is held fixed by another force as shown in the opposite Figure. The stress in this case is called a *shear stress*. If the object is originally a rectangular block, a shear stress results in a shape whose cross section is a parallelogram. A book pushed sideways as shown in the following Figure is an example of an object subjected to a shear stress. To a first approximation (for small distortions), no change in volume occurs with this deformation.





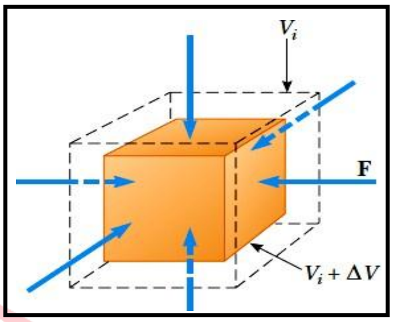
We define the **shear stress** as F/A, the ratio of the tangential force to the area A of the face being sheared. The **shear strain** is defined as the ratio $\Delta x/h$, where Δx is the horizontal distance that the sheared face moves and h is the height of the object. In terms of these quantities, the **shear modulus** is

$$S = \frac{\text{shear stress}}{\text{shear strain}} = \frac{F/A}{\Delta x/h} \qquad (2.5)$$

Like Young's modulus, the unit of shear modulus is the ratio of that for force to that for area.

2.4.3 Bulk Modulus: Volume Elasticity

Bulk modulus characterizes the response of an object to changes in a force of uniform magnitude applied perpendicularly over the entire surface of the object as shown in the opposite Figure. (We assume here the object is made of a single substance.) An object subject to this type of deformation undergoes a change in volume but no change in shape. The **volume stress** is defined as the ratio of the magnitude of the total force F exerted on a surface to the area A of the surface. The quantity $P = F/A$ is called **pressure**, which we shall study in more detail in Chapter 3. If the pressure on an object changes by an amount $\Delta P = \Delta F/A$, the object experiences a volume change ΔV. The volume strain is equal to the change in volume ΔV divided by the initial volume V_i. Therefore, from Equation 2.1, we can characterize a volume ("bulk") compression in terms of the **bulk modulus**, which is defined as



$$B = \frac{\text{volume stress}}{\text{volume strain}} = -\frac{\Delta F/A}{\Delta V/V_i} = -\frac{\Delta P}{\Delta V/V_i} \qquad (2.6)$$

A negative sign is inserted in this defining equation so that B is a positive number. This maneuver is necessary because an increase in pressure (positive ΔP) causes a decrease in volume (negative ΔV) and vice versa.

The reciprocal of the bulk modulus is called the **compressibility** of the material and is denoted by k:

$$k = \frac{1}{B} = -\frac{\Delta V/V_i}{\Delta P} \qquad (2.7)$$

Compressibility is the fractional decrease in volume, $-\Delta V / V_i$ per unit increase in pressure ΔP. The units of compressibility are those of *reciprocal pressure*, Pa^{-1} or atm^{-1}.

Application: Bulk Stress on an Anglerfish The anglerfish (Melanocetus johnsoni) is found in oceans throughout the world at depths as great as 1000 m, where the pressure (that is, the bulk stress) is about 100 atmospheres. Anglerfish are able to withstand such stress because they have no internal air spaces, unlike fish found in the upper ocean where pressures are lower. The largest anglerfish are about 12 cm long.

Typical values of the elastic moduli for some representative materials are given in Table 2.1. If you look up such values in a different source, you may find the reciprocal of the bulk modulus listed.

Table 2.1 Typical Values for Elastic Moduli

Substance	Young's Modulus (N/m^2)	Shear Modulus (N/m^2)	Bulk Modulus (N/m^2)
Tungsten	35×10^{10}	14×10^{10}	20×10^{10}
Steel	20×10^{10}	8.4×10^{10}	6×10^{10}
Copper	11×10^{10}	4.2×10^{10}	14×10^{10}
Brass	9.1×10^{10}	3.5×10^{10}	6.1×10^{10}
Aluminum	7.0×10^{10}	2.5×10^{10}	7.0×10^{10}
Glass	$6.5–7.8 \times 10^{10}$	$2.6–3.2 \times 10^{10}$	$5.0–5.5 \times 10^{10}$
Quartz	5.6×10^{10}	2.6×10^{10}	2.7×10^{10}
Water	—	—	0.21×10^{10}
Mercury	—	—	2.8×10^{10}

Notice from Table 2.1 that both solids and liquids have a bulk modulus. No shear modulus and no Young's modulus are given for liquids, however, because a liquid does not sustain a shearing stress or a tensile stress. If a shearing force or a tensile force is applied to a liquid, the liquid simply flows in response.

Quick Quiz 2.2

A block of iron is sliding across a horizontal floor. The friction force between the block and the floor causes the block to deform. To describe the relationship between stress and strain for the block, you would use (a) Young's modulus

(b) shear modulus (c) bulk modulus (d) none of these.

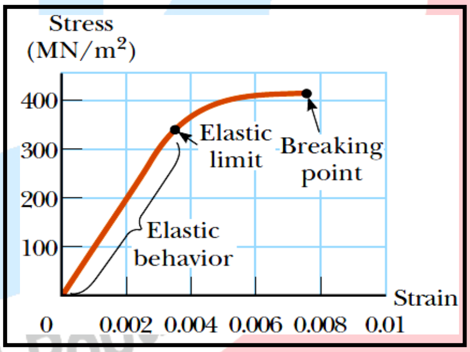
Quick Quiz 2.3

Spacecraft carries a steel sphere to a planet on which atmospheric pressure is much higher than on the Earth. The higher pressure causes the radius of the sphere to decrease. To describe the relationship between stress and strain for the sphere, you would use (a) Young's modulus (b) shear modulus (c) bulk modulus

(d) none of these.

2.6 Stress-Strain Curve

For relatively small tensile stresses, the material returns to its initial length when the force is removed. The **elastic limit** of a substance is defined as the maximum stress that can be applied to the substance before it becomes permanently deformed and does not return to its initial length. It is possible to exceed the elastic limit of a substance by applying a sufficiently large stress as seen in the opposite Figure. Initially, a stress-versus strain curve is a straight line. As the stress increases, however, the curve is no longer a straight line. When the stress exceeds the elastic limit, the object is permanently deformed and does not return to its original shape after the stress is removed.

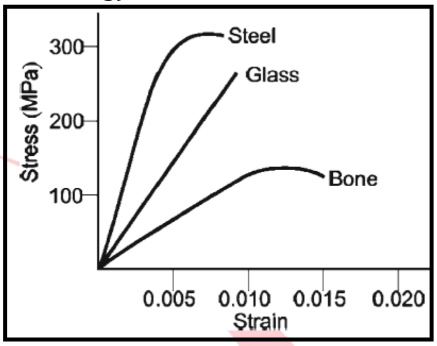


The region from the origin to the elastic limit is called the ***elastic region***. If the object is stretched beyond the elastic limit, it enters the ***plastic region***. As the stress is increased even further, the material ultimately breaks. The maximum elongation is reached at the *breaking point*. The maximum force that can be applied without breaking is called the **ultimate strength** of the material.

If the stress in a material is directly proportional to the strain for strains up to the elastic limit, the material is called a Hookean material.

The mineral content of bone affects its mechanical property. Higher mineralization makes the bone stronger and stiffer (higher modulus of elasticity), but it lowers the toughness; that is, it is less capable of absorbing shock and strain energy.

Bone shows a linear range in which the stress increases in proportion to the strain. The slope of this region is defined as Young's modulus, or the elastic modulus. An illustration of the material properties of bone relative to other materials is shown in the opposite Figure.



Example 2.2 Squeezing a Brass Sphere

A solid brass sphere is initially surrounded by air, and the air pressure exerted on it is 1×10^5 N/m^2 (normal atmospheric pressure). The sphere is lowered into the ocean to a depth where the pressure is 2×10^7 N/m^2. The volume of the sphere in air is 0.5 m^3. By how much does this volume change once the sphere is submerged?

Solution

We perform a simple calculation involving Equation 2.4, so we categorize this example as a substitution problem.

Solve Equation 2.4 for the volume change of the sphere:

$$\Delta V = -\frac{V_i \Delta P}{B}$$

Substitute numerical values:

$$\Delta V = -\frac{(0.50\ \mathrm{m^3})(2.0\times10^7\ \mathrm{N/m^2} - 1.0\times10^5\ \mathrm{N/m^2})}{6.1\times10^{10}\ \mathrm{N/m^2}}$$

$$= -1.6\times10^{-4}\ \mathrm{m^3}$$

The negative sign indicates that the volume of the sphere decreases.

2.7 Fracture

If the stress on a solid object is so great, the object fractures or breaks.

Table 2.2 lists the ultimate tensile strength, compressive strength, and shear strength for a variety of materials. These values give the maximum force per unit area that an object can withstand under each of these three types of stress. They are, however, representative values only, and the actual value for a given specimen can differ considerably. It is therefore necessary to maintain a "**safety factor**" of from 3 to perhaps 10 or more- that is, the actual stresses on a structure should not exceed one-tenth to one-third of the values given in Table 2.2. You may encounter tables of the "allowable stresses" in which appropriate safety factors have already been included.

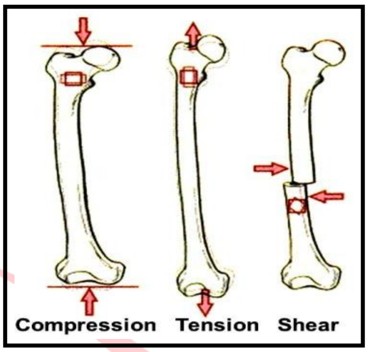


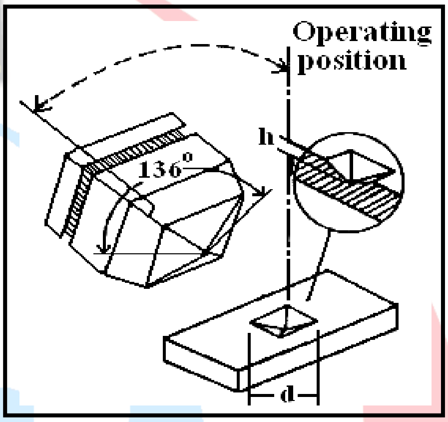
Table 2.2 Ultimate Strengths of some Materials.

Material	Tensile Strength (N/m^2)	Compressive Strength (N/m^2)	Shear Strength (N/m^2)
Steel	500×10^6	500×10^6	250×10^6
Brass	250×10^6	250×10^6	200×10^6
Aluminum	200×10^6	200×10^6	200×10^6
Concrete	2×10^6	20×10^6	2×10^6
Bone (limb)	130×10^6	170×10^6	

2.8 Hardness

One of the most important properties of dental materials is hardness. The hardness of a material is defined as its resistance to permanent or plastic deformation. It represents the resistance of a material to penetration of another much harder indenter. There are different procedures for testing the hardness, and in dentistry the most common are Knoop and Vickers methods. In this case, the hardness is expressed by Knoop (HKN) or Vickers (HVN) number.

In the Vickers method, which is adopted in this investigation, a square- base diamond pyramid of face angle 136° is pressed into the specimen surface as shown in the opposite Figure. Because of the shape of the indenter, this is frequently called the diamond-pyramid hardness test.



The Vickers hardness number (H_V) is obtained as the ratio of the applied load to the area of the resulting indentation. With the given pyramid geometry the H_V is expressed by:

$$H_V = \frac{2P\sin\frac{\theta}{2}}{d^2} \times 1000 = 1854.4\frac{P}{d^2} \qquad (2.7)$$

Where

H_V : Vickers hardness number (kgf/mm^2)

P : Test load on diamond indenter (gf)

θ : Opposite face angle of the tip of diamond indenter (136°)

d : Diagonal length of the indentation on the specimen's surface (μm).

Equation (2.7) is used for calculating the H_V after measuring the average value of the diagonal length of the pyramids formed on the specimen's surface.

The Vickers hardness test has received fairly wide acceptance for practical work because it provides a continuous scale of hardness, for a given load, from very soft materials with a H_V of 5 to extremely hard materials with a H_V of 1500.

Today, of course, advances in technology, enable the production of alloys with improved properties. Taking into account all metallic materials, which have so far been applied in dentistry and which are currently in use, the biggest advantage may be given to titanium and titanium based alloys due to their superior properties, the economy factor and the most important fact that they are not harmful to the patient.

One of the major challenges in the development of novel dental materials is to produce biomaterials exhibiting mechanical properties able to match those of the tooth hard structures, i.e., dentine and enamel. Table 2.2 summarizes the major mechanical features of a tooth. In general, the tooth mechanical properties are dependent on the patient's age; therefore, the choice of one particular material for dental applications rather than another one should ideally be done by taking into account this parameter.

Table 2.2 Mechanical Properties of a Natural Tooth.

Property	Dentine	Enamel
Bending strength (MPa)	30-120	60-200
Elastic modulus (GPa)	18-26	70-100
Hardness (GPa)	0.7-0.8	3.0-5.5

Problems

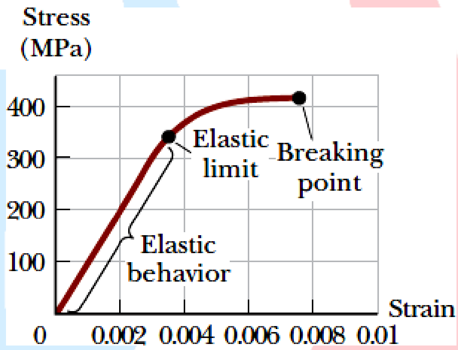
1. A 200-kg load is hung on a wire having a length of 4 m, cross-sectional area 0.2×10^{-4} m^2, and Young's modulus 8×10^{10} N/m^2. **What** is its increase in length?

---

2. Assume that Young's modulus is 1.5×10^{10} N/m^2 for bone and that the bone will fracture if stress greater than 1.5×10^{8} N/m^2 is imposed on it. (a) **What** is the maximum force that can be exerted on the femur bone in the leg if it has a minimum effective diameter of 2.5 cm? (b) If this much force is applied compressively, **by how much** does the 25-cm-long bone shorten?

---

3. **Evaluate** Young's modulus for the material whose stress versus-strain curve is shown in the opposite Figure.



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4. A child slides across a floor in a pair of rubber-soled shoes. The friction force acting on each foot is 20 N. The footprint area of each shoe sole is 14 cm^2, and the thickness of each sole is 5 mm. **Find** the horizontal distance by which the upper and lower surfaces of each sole are offset. The shear modulus of the rubber is 3 MN/m^2.

---

5. If the shear stress in steel exceeds 4×10^{8} N/m^2, the steel ruptures. **Determine**

the shearing force necessary to (a) shear a steel bolt 1 cm in diameter and

(b) punch a 1-cm-diameter hole in a steel plate 0.5 cm thick.

---

6. When water freezes, it expands by about 9%. **What** pressure increase would occur inside your automobile engine block if the water in it froze? (The bulk modulus of ice is 2×10^9 N/m^2.)

7. The deepest point in the ocean is in the Mariana Trench, about 11 km deep. The pressure at this depth is huge, about 1.13×10^8 N/m^2. (a) **Calculate** the change in volume of 1 m^3 of seawater carried from the surface to this deepest point in the Pacific ocean. (b) The density of seawater at the surface is 1.03×10^3 kg/m^3. **Find** its density at the bottom. The bulk modulus of water is 0.21×10^{10} N/m^2.

---

8. A 1.6 m-long steel piano wire has a diameter of 0.2 cm. **How** great is the tension in the wire if it stretches 0.3 cm when tightened?

---

9. a small elevator with a mass of 550 kg hang from a steel cable that is 3 m long when not loaded. The wires making up the cable have a total cross -sectional area of 0.2 cm^2, and with a 550 kg load, the cable stretches 0.4 cm beyond its unloaded length. **Calculate**: (a) the cable's stress, (b) the cable's stress and

   (c) the value of Young's modulus for the cable's steel.

---

10. the bulk modulus of water is 2.1×10^9 N/m^2. **By how mush** does a cubic meter of water decrease in volume when it is taken from surface of the ocean down to a depth 1 km, where the pressure is 9.8×10^6 N/m^2 greater than at the surface?

---

11. A fluid with an initial volume of 0.35 m3 is subjected to a pressure decrease of 3.2×10^3 N/m^2. The volume is then found to have increased by 0.2 cm^3. **What** is the bulk modulus of this fluid?

---

12. The femur bone in the leg has a minimum effective cross section of about 3 cm^2. **How much** compressive force can it withstand before breaking? The ultimate compressive strength of bone is 170×10^6 N/m^2.

13. If a compressive force of 3×10^4 N is exerted on the end of a 20 cm-long bone of cross-sectional area 3.6 cm^2, (a) **will** the bone break, and (b) if not, **by how much** does it shorten? The ultimate compressive strength of bone is 170×10^6 N/m^2. Young's modulus of bone is 15×10^9 N/m^2.

---

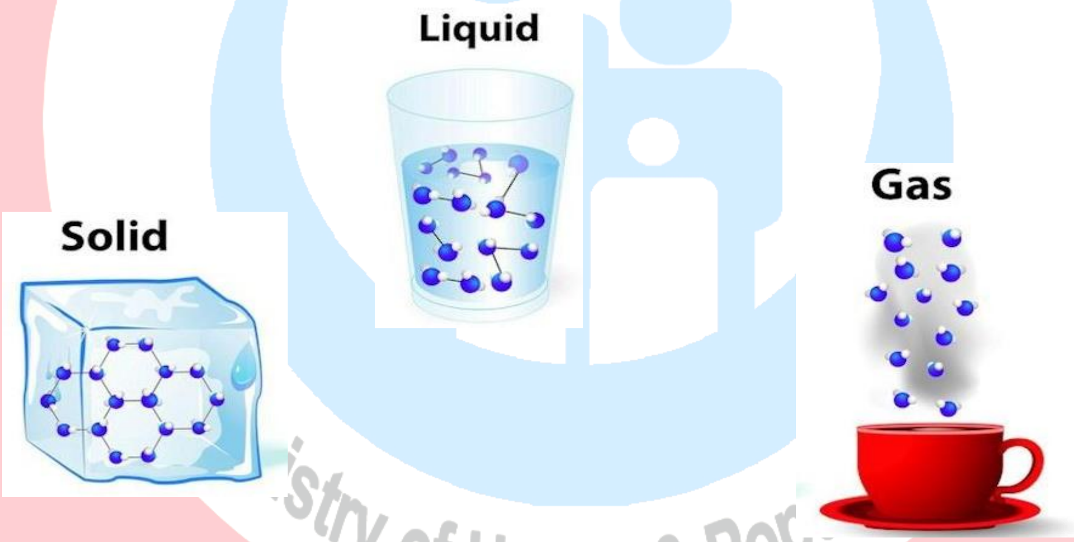
14. A steel cable is to support an elevator whose total (loaded) mass is not to exceed 3100 kg. If the maximum acceleration of the elevator is 1.2 m/s^2, **calculate** the diameter of cable required. Assume a safety factor of 7. The ultimate compressive strength of steel is 500×10^6 N/m^2.

---

15. **What** pressure must you exert on a sample of water if you want to compress its volume by 0.2%? the bulk modulus of water is 0.21×10^{10} N/m^2

Chapter (3) Fluid Statics

Matter is normally classified as being in one of three states: solid, liquid, or gas. From everyday experience, we know that a solid has a definite volume and shape. A brick maintains its familiar shape and size day in and day out. We also know that a liquid has a definite volume but no definite shape. Finally, we know that an unconfined gas has neither a definite volume nor a definite shape. These descriptions help us picture the states of matter, but they are somewhat artificial. For example, asphalt and plastics are normally considered solids, but over long periods of time they tend to flow like liquids. Likewise, most substances can be a solid, a liquid, or a gas (or a combination of any of these), depending on the temperature and pressure. In general, the time it takes a particular substance to change its shape in response to an external force determines whether we treat the substance as a solid, a liquid, or a gas. Since liquids and gases do not maintain a fixed shape, they both have the ability to flow; they are thus often referred to collectively as **fluids**.



The division of matter into three states in not always simple. How, for example, should butter be classified? Furthermore, a fourth state of matter can be distinguished, the **plasma** state, which occurs only at very high temperatures and consists of ionized atoms (electrons separated from the nuclei).

Fluids play a vital role in many aspects of everyday life. We drink them, breathe them, swim in them. They circulate through our bodies and control our weather. Airplanes fly through them; ships float in them. A fluid is any substance that can flow; we use the term for both liquids and gases. We usually think of a gas as easily compressed and a liquid as nearly incompressible, although there are exceptional cases.

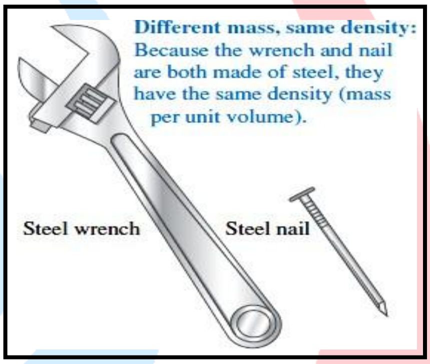
The term "**fluid statics**" is applied to the study of fluids at rest, while the term "**fluid dynamics**" is applied to the study of fluids in motion. First, we consider the mechanics of a fluid at rest—that is, *fluid statics*. We then treat the mechanics of fluids in motion—that is, *fluid dynamics* in the next chapter.

3.1 Density

An important property of any material is its **density**, defined as *its mass per unit volume*. A homogeneous material such as ice or iron has the same density throughout. We use (the Greek letter rho) for density. If a mass *m* of homogeneous material has volume *V*, the density ρ is

$$\rho = \frac{m}{V} \tag{3.1}$$

Density is a characteristic property of any pure substance. Thus, two objects made of the same material have the same density even though they may have different masses and different volumes. That's because the ratio of mass to volume is the same for both objects (as shown in the opposite Figure).



The SI unit of density is the kilogram per cubic meter (1 kg/m^3). The cgs unit, the gram per cubic centimeter (1 g/cm^3), is also widely used:

$$1\ \text{g/cm}^3 = 1000\ \text{kg/m}^3$$

Aluminum, for example, has a density of 2.70×10^3 kg/m^3, and iron has a density of 7.86×10^3 kg/m^3. An extreme difference in density can be imagined by thinking about holding a 10-centimeter (cm) cube of Styrofoam in one hand and a 10-cm cube of lead in the other. A list of densities for various substances is given in Table 3.1. This Table specifies temperature and pressure because they affect the density of substances (although the effect is slight for liquids and solids).

Table 3.1

Densities of Some Common Substances at Standard Temperature (0°C) and Pressure (Atmospheric)			
Substance	$\rho\,(kg/m^3)$	Substance	$\rho\,(kg/m^3)$
Air	1.29	Ice	0.917×10^3
Aluminum	2.70×10^3	Iron	7.86×10^3
Benzene	0.879×10^3	Lead	11.3×10^3
Copper	8.92×10^3	Mercury	13.6×10^3
Ethyl alcohol	0.806×10^3	Oak	0.710×10^3
Fresh water	1.00×10^3	Oxygen gas	1.43
Glycerin	1.26×10^3	Pine	0.373×10^3
Gold	19.3×10^3	Platinum	21.4×10^3
Helium gas	1.79×10^{-1}	Seawater	1.03×10^3
Hydrogen gas	8.99×10^{-2}	Silver	10.5×10^3

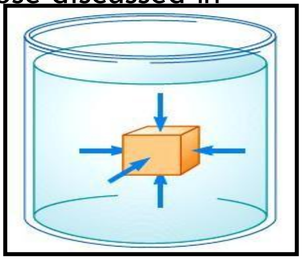
The density of some materials varies from point to point within the material. One example is the material of the human body, which includes low-density fat (about 940 kg/m^3) and high-density bone (from 1700 to 2500 kg/m^3). Two others are the earth's atmosphere (which is less dense at high altitudes) and oceans (which are denser at greater depths). For these materials, Equation (3.1) describes the **average density**. In general, the density of a material depends on environmental factors such as temperature and pressure.

Quick Quiz 1.1

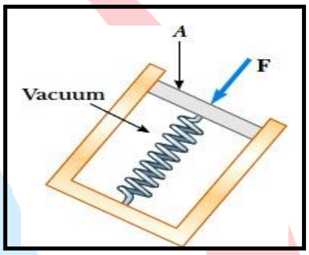
In a machine shop, two cams are produced, one of aluminum and one of iron. Both cams have the same mass. Which cam is larger? (a) The aluminum cam is larger. (b) The iron cam is larger. (c) Both cams have the same size.

3.2 Pressure in a Fluid

Fluids do not sustain shearing stresses or tensile stresses such as those discussed in Chapter 2; therefore, the only stress that can be exerted on an object submerged in a static fluid is one that tends to compress the object from all sides. In other words, the force exerted by a static fluid on an object is always perpendicular to the surfaces of the object as shown in the opposite Figure.



The pressure in a fluid can be measured with the device pictured in the opposite Figure. The device consists of an evacuated cylinder that encloses a light piston connected to a spring. As the device is submerged in a fluid, the fluid presses on the top of the piston and compresses the spring until the inward force exerted by the fluid is balanced by the outward force exerted by the spring. The fluid pressure can be measured directly if the spring is calibrated in advance. If F is the magnitude of the force exerted on the piston and A is the surface area of the piston, then the pressure P of the fluid at the level to which the device has been submerged is defined as the ratio F/A:



$$P = \frac{F}{A} \qquad (3.2)$$

Note that pressure is a scalar quantity because it is proportional to the magnitude of the force on the piston.

If the pressure varies over an area, we can evaluate the infinitesimal force dF on an infinitesimal surface element of area dA as

$$dF = PdA \qquad (3.3)$$

where P is the pressure at the location of the area dA. To calculate the total force exerted on a surface of a container, we must integrate Equation 3.2 over the surface.

The units of pressure are newtons per square meter (N/m^2) in the SI system. Other units sometimes used are dyne/cm^2 and lb/in^2 (sometimes abbreviated "psi"). Another name for the SI unit of pressure is the **pascal** (Pa):

$$\mathbf{1\ Pa = 1\ N/m^2}$$

As an example of calculating pressure, a 60-kg person whose two feet cover an area of 500 cm^2 will exert a pressure of

$$\frac{F}{A} = \frac{mg}{A} = \frac{(60\ kg)(9.8\ m/s^2)}{(500 \times 10^{-4}\ m^2)} \approx 12 \times 10^3\ N/m^2$$

on the ground. If the person stands on one foot, the is the same but the area will be half, so the pressure will be twice as much: $24{\times}10^3\ N/m^2$.

For a tactile demonstration of the definition of pressure, hold a tack between your thumb and forefinger, with the point of the tack on your thumb and the head of the tack on your forefinger. Now gently press your thumb and forefinger together. Your thumb will begin to feel pain immediately while your forefinger will not. The tack is exerting the same force on both your thumb and forefinger, but the pressure on your thumb is much larger because of the small area over which the force is applied.

Quick Quiz 3.1

Suppose you are standing directly behind someone who steps back and accidentally stomps on your foot with the heel of one shoe. Would you be better off if that person were (a) a large professional basketball player wearing sneakers

(b) a petite woman wearing spike-heeled shoes?

Quick Quiz 3.2

A closed, empty soda bottle, with a diameter of 10 cm at its base and 2 cm at its top, is lying sideways on a desk in the air. What is the ratio of the value of the external pressure at its base to the value of the pressure at its cap?

(a) 1 (b) 5 (c) 1/5 (d) 25 (e) 1/25

Example 3.1 The Water Bed

The mattress of a water bed is 2 m long by 2 m wide and 30 cm deep.

(A) Find the weight of the water in the mattress.

(B) Find the pressure exerted by the water bed on the floor when the bed rests in its normal position. Assume the entire lower surface of the bed makes contact with the floor.

(C) What if the water bed is replaced by a 300-lb ordinary bed that is supported by four legs? Each leg has a circular cross section of radius 2 cm. What pressure does this bed exert on the floor?

Solution

(A) The weight of the water in the mattress:

Find the volume of the water filling the mattress:

$$V = (2\ \mathrm{m})(2\ m)(0.3\ m) = 1.2\ m^3$$

Use Equation 3.1 and the density of fresh water (10^3 kg/m^3) to find the mass of the water bed:

$$M = \rho V = (10^3\ \mathrm{kg/m^3})(1.2\ m^3) = 1200\ kg$$

Find the weight of the bed:

$$Weight = Mg$$

$$Weight = (1200\ kg)(9.8\ m/s^2) = 1.18 \times 10^4 N$$

(B) The pressure exerted by the water bed on the floor:

When the water bed is in its normal position, the area in contact with the floor is

$$\mathrm{A} = (2\ \mathrm{m})(2\ \mathrm{m}) = 4\ \mathrm{m}^2$$

Use Equation 3.2 to find the pressure:

$$P = \frac{F}{A} = \frac{1.18 \times 10^4\ N}{4\ m^2} = 2.95 \times 10^3\ N/m^2$$

So, the pressure exerted by the water bed on the floor is **2.95×10^3Pa**

(C) The weight of the regular bed is distributed over four circular cross sections at the bottom of the legs. Therefore, the pressure is

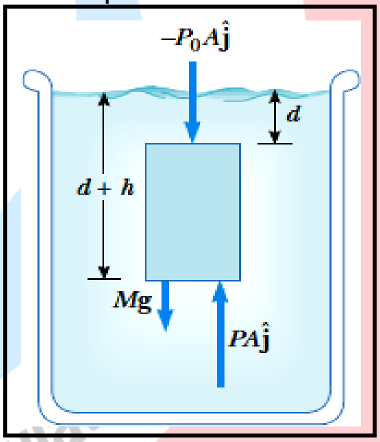
$$P = \frac{F}{A} = \frac{Mg}{4\ (\pi r^2)} = \frac{300\ Ib}{4\pi(0.02\ m)^2}\left(\frac{1\ N}{0.225\ Ib}\right)$$
$$= 2.65 \times 10^5\ N/m^2$$

This result is almost 100 times larger than the pressure due to the water bed! The weight of the regular bed, even though it is much less than the weight of the water bed, is applied over the very small area of the four legs. The high pressure on the floor at the feet of a regular bed could cause dents in wood floors or permanently crush carpet pile.

3.3 Variation of Pressure with Depth

As divers well know, water pressure increases with depth. Likewise, atmospheric pressure decreases with increasing altitude; for this reason, aircraft flying at high altitudes must have pressurized cabins for the comfort of the passengers.

We now show how the pressure in a liquid increases with depth. As Equation 1.1 describes, the *density* of a substance is defined as its mass per unit volume; Table 1.1 lists the densities of various substances. These values vary slightly with temperature because the volume of a substance is dependent on temperature. Under standard conditions (at 0ºC and at atmospheric pressure), the densities of gases are about 1/1000 the densities of solids and liquids. This difference in densities implies that the average molecular spacing in a gas under these conditions is about ten times greater than that in a solid or liquid.



Now consider a liquid of density ρ at rest as shown in the opposite Figure. We assume that ρ is uniform throughout the liquid; this means that the liquid is incompressible. Let us select a sample of the liquid contained within an imaginary cylinder of cross- sectional area A extending from depth d to depth d + h. The liquid external to our sample exerts forces at all points on the surface of the sample, perpendicular to the surface. The pressure exerted by the liquid on the bottom face of the sample is P, and the pressure on the top face is P_0. Therefore, the upward force exerted by the outside fluid on the bottom of the cylinder has a magnitude PA, and the downward force exerted on the top has a magnitude P_0A. The mass of liquid in the cylinder is $M = \rho V = \rho Ah$; therefore, the weight of the liquid in the cylinder is $Mg = \rho Ahg$. Because the cylinder is in equilibrium, the net force acting on it must be zero. Choosing upward to be the positive y direction, we see that

$$\sum \mathbf{F} = PA\hat{\mathbf{j}} - P_0A\hat{\mathbf{j}} - M\mathbf{g}\hat{\mathbf{j}} = 0$$

or

$$PA - P_0A - \rho Ahg = 0$$

$$P = P_0 + \rho gh \qquad (3.4)$$

That is, **the pressure P at a depth h below a point in the liquid at which the pressure is P_0 is greater by an amount ρgh**. If the liquid is open to the atmosphere and P_0 is the pressure at the surface of the liquid, then P_0 is atmospheric pressure. In our calculations and working of end-of-chapter problems, we usually take atmospheric pressure to be

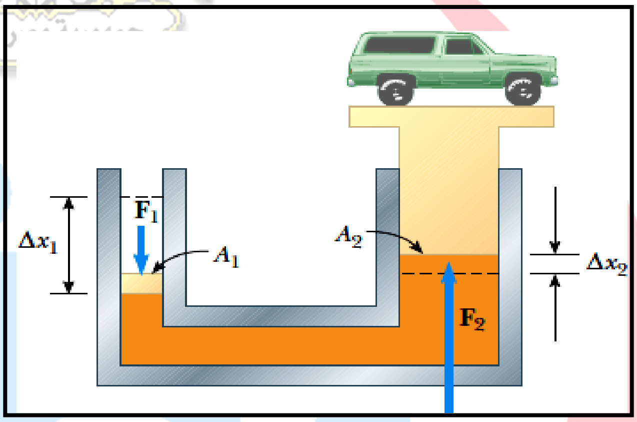
$$P_0 = 1\ \text{atm} = 1.013 \times 10^5\ \text{Pa}$$

Equation 3.3 implies that the pressure is the same at all points having the same depth, independent of the shape of the container.

3.4 Pascal's Principle

Because the pressure in a fluid depends on depth and on the value of P_0, any increase in pressure at the surface must be transmitted to every other point in the fluid. This concept was first recognized by French scientist Blaise Pascal (1623– 1662) and is called **Pascal's law: a change in the pressure applied to a fluid is transmitted undiminished to every point of the fluid and to the walls of the container.**

An important application of Pascal's law is the hydraulic press illustrated in the opposite Figure. A force of magnitude F_1 is applied to a small piston of surface area A_1. The pressure is transmitted through an incompressible liquid to a larger piston of surface area A_2. Because the pressure must be the same on both sides,



$P = F_1 / A_1 = F_2/A_2$. Therefore, the force F_2 is greater than the force F_1 by a factor A_2/A_1. By designing a hydraulic press with appropriate areas A_1 and A_2, a large output force can be applied by means of a small input force. Hydraulic brakes, car lifts, hydraulic jacks, and forklifts all make use of this principle as shown in the following Figure.

Because liquid is neither added nor removed from the system, the volume of liquid pushed down on the left in Figure (page 29) as the piston moves downward through a displacement Δx_1 equals the volume of liquid pushed up on the right as the right piston moves upward through a displacement Δx_2. That is, $A_1 \Delta x_1 = A_2 \Delta x_2$; thus, $A_2/A_1 = \Delta x_1/\Delta x_2$. We have already shown that $A_2/A_1 = F_2/F_1$. Thus, $F_2/F_1 = \Delta x_1/\Delta x_2$, so $\mathbf{F_1 \Delta x_1 = F_2 \Delta x_2}$. Each side of this equation is the work done by the force. Thus, the work done by $\mathbf{F}_1$ on the input piston equals the work done by $\mathbf{F}_2$ on the output piston, as it must in order to conserve energy.

Quick Quiz 3.3

The pressure at the bottom of a filled glass of water (ρ =1000 kg/m^3) is P. The water is poured out and the glass is filled with ethyl alcohol (ρ =806 kg/m^3). The pressure at the bottom of the glass is (a) smaller than P (b) equal to P (c) larger than P (d) indeterminate.

Example 3.2 The Car Lift

In a car lift used in a service station, compressed air exerts a force on a small piston that has a circular cross section of radius 5 cm. This pressure is transmitted by a liquid to a piston that has a radius of 15 cm.

(A) What force must the compressed air exert to lift a car weighing 13300 N?

(B) What air pressure produces this force?

Solution

(A) Because the pressure exerted by the compressed air is transmitted undiminished throughout the liquid, we have

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

Solve this equation for F_1:

$$F_1 = \left(\frac{A_1}{A_2}\right) F_2 = \left(\frac{\pi(5 \times 10^{-2} m)^2}{\pi(15 \times 10^{-2} m)^2}\right)(13300\ N)$$

$$F_1 = 1.48 \times 10^3\ \mathrm{N}$$

(B) The air pressure produces this force is

$$P = \frac{F_1}{A_1} = \frac{1.48 \times 10^3\ N}{\pi(5 \times 10^{-2} m)^2} = 1.88 \times 10^5\ N/m^2$$

This pressure is approximately twice atmospheric pressure.

Example 3.3 A Pain in Your Ear

Estimate the force exerted on your eardrum due to the water above when you are swimming at the bottom of a pool that is 5 m deep.

Solution

As you descend in the water, the pressure increases. You may have noticed this increased pressure in your ears while diving in a swimming pool, a lake, or the ocean. We can find the pressure difference exerted on the eardrum from the depth given in the problem; then, after estimating the ear drum's surface area, we can determine the net force the water exerts on it.

The air inside the middle ear is normally at atmospheric pressure P_0. Therefore, to find the net force on the eardrum, we must consider the difference between the total pressure at the bottom of the pool and atmospheric pressure. Let's estimate the surface area of the eardrum to be approximately 1 cm^2 = 1 ×10^{-4} m^2.

Use Equation 3.3 to find this pressure difference:

$$P_{bot} - P_0 = \rho g h$$

$$= (10^3 kg/m^3)(9.8\, m/s^2)(5\, m) = 4.9 \times 10^4\ N/m^2$$

Use Equation 3.1 to find the magnitude of the net force on the ear:

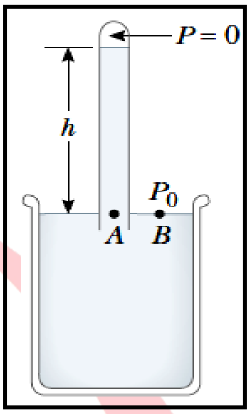
$$F = (P_{bot} - P_0)A = (4.9 \times 10^4\ N/m^2)(1 \times 10^{-4}\ m^2) \approx 5\ N$$

Because a force of this magnitude on the eardrum is extremely uncomfortable, swimmers often "pop their ears" while under water, an action that pushes air from the lungs into the middle ear. Using this technique equalizes the pressure on the two sides of the eardrum and relieves the discomfort.

3.5 Pressure Measurements

During the weather report on a television news program, the ***barometric pressure*** is often provided. This reading is the current local pressure of the atmosphere, which varies over a small range from the standard value provided earlier. How is this pressure measured?

One instrument used to measure atmospheric pressure is the common **barometer**, invented by Evangelista Torricelli (1608-1647). A long tube closed at one end is filled with mercury and then inverted into a dish of mercury as shown in the opposite Figure. The closed end of the tube is nearly a vacuum, so the pressure at the top of the mercury column can be taken as zero. In the opposite Figure, the pressure at point *A*, due to the column of mercury, must equal the pressure at point *B*, due to the atmosphere. If that were not the case, there would be a net force that would move mercury from one point to the other until equilibrium is established. Therefore, $P_0 = \rho_{Hg} g h$, where ρ_{Hg} is the density of the mercury and h is the height of the mercury column. As atmospheric pressure varies, the height of the mercury column varies, so the height can be calibrated to measure atmospheric pressure.



Let us determine the height of a mercury column for one atmosphere of pressure, P_0 = 1 atm = 1.013 ×10^5 Pa:

$$P_0 = \rho_{Hg} g h \rightarrow h = \frac{P_0}{\rho_{Hg} g}$$

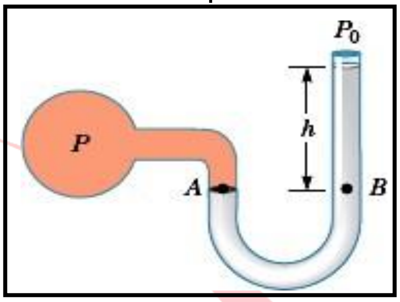
$$\therefore h = \frac{1.013 \times 10^5 \ N/m^2}{(13.6 \times 10^3 kg/m^3)(9.8 \ m/s^2)} = 0.76 \ m$$

Based on such a calculation, one atmosphere of pressure is defined to be the pressure equivalent of a column of mercury that is exactly 0.76 m in height at 0ºC. Thus the mercury barometer reads the atmospheric pressure P_0 directly from the height of the mercury column.

Pressures are often described in terms of the height of the corresponding mercury column, as so many “millimeters of mercury” (abbreviated mm Hg).

A pressure of 1 mm Hg is called 1 *torr*, after Evangelista Torricelli, inventor of the mercury barometer. But these units depend on the density of mercury, which varies with temperature, and on the value of g, which varies with location, so the pascal is the preferred unit of pressure.

A device for measuring the pressure of a gas contained in a vessel is the open-tube manometer illustrated in the opposite Figure. One end of a U-shaped tube containing a liquid is open to the atmosphere, and the other end is connected to a system of unknown pressure P. The pressures at points A and B must be the same (otherwise, the curved portion of the liquid would experience a net force and would accelerate), and the pressure at A is the unknown pressure of the gas. Therefore, equating the unknown pressure P to the pressure at point B, we see that $P = P_0 + \rho gh$. The difference in pressure $P - P_0$ is equal to ρgh. The pressure P is called the **absolute pressure**, while the difference $P - P_0$ is called the **gauge pressure** P_G. For example, the pressure you measure in your bicycle tire is gauge pressure. Thus, to get the absolute pressure P, one must add the atmospheric pressure P_0, to the gauge pressure P_G:



$$P = P_0 + P_G$$

Application: Gauge Pressure of Blood

Blood-pressure readings, such as 130/80, give the maximum and minimum gauge pressures in the arteries, measured in mm Hg or torr.

Blood pressure varies with vertical position within the body; the standard reference point is the upper arm, level with the heart.

Quick Quiz 3.3

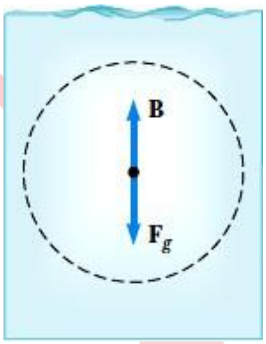
Several common barometers are built, with a variety of fluids. For which of the following fluids will the column of fluid in the barometer be the highest?

(a) mercury (b) water (c) ethyl alcohol (d) benzene

3.6 Buoyant Forces and Archimedes's Principle

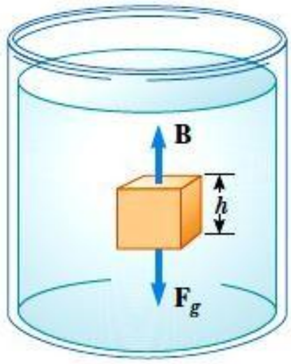
Have you ever tried to push a beach ball down under water (see the opposite Figure)? It is extremely difficult to do because of the large upward force exerted by the water on the ball. The upward force exerted by a fluid on any immersed object is called a **buoyant force**.

We can determine the magnitude of a buoyant force by applying some logic. Imagine a beach ball-sized parcel of water beneath the water surface as shown in the opposite Figure. Because this parcel is in equilibrium, there must be an upward force that balances the downward gravitational force on the parcel. This upward force is the buoyant force, and its magnitude is equal to the weight of the water in the parcel. The buoyant force is the resultant force on the parcel due to all forces applied by the fluid surrounding the parcel.



Now imagine replacing the beach ball-sized parcel of water with a beach ball of the same size. The net force applied by the fluid surrounding the beach ball is the same, regardless of whether it is applied to a beach ball or to a parcel of water. Consequently, **the magnitude of the buoyant force on an object always equals the weight of the fluid displaced by the object**. This statement is known as **Archimedes's principle**.

To understand the origin of the buoyant force, consider a cube immersed in a liquid as in the opposite Figure. The pressure P_{bot} at the bottom of the cube is greater than the pressure P_{top} at the top by an amount $\rho_{fluid}gh$, where h is the height of the cube and ρ_{fluid} is the density of the fluid. The pressure at the bottom of the cube causes an upward force equal to $P_{bot}A$, where A is the area of the bottom face. The pressure at the top of the cube causes a downward force equal to $P_{top}A$. The resultant of these two forces is the buoyant force B:



$$B = (P_{bot} - P_{top})A = (\rho_{fluid} g h)A$$

$$B = \rho_{\text{fluid}} g V_{\text{disp}} \quad (3.5)$$

where $V_{\text{disp}} = Ah$ is the volume of the fluid displaced by the cube. Because the product $\rho_{\text{fluid}} V_{\text{disp}}$ is equal to the mass of fluid displaced by the object,

$$B = Mg$$

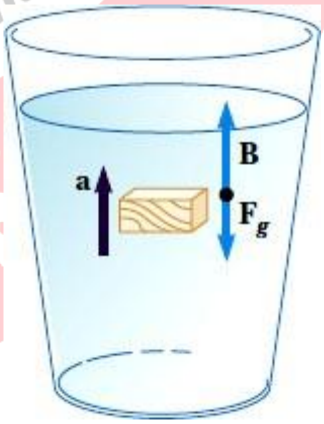
where Mg is the weight of the fluid displaced by the cube. This result is consistent with our initial statement about Archimedes's principle above, based on the discussion of the beach ball.

Before we proceed with a few examples, it is instructive to discuss two common situations: a totally submerged object and a floating (partly submerged) object.

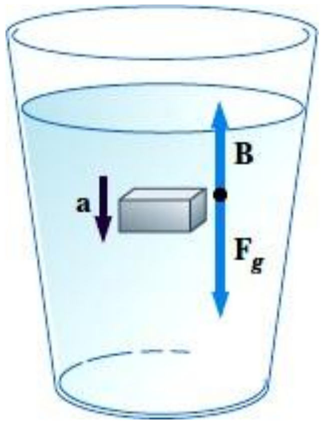
Case 1: Totally Submerged Object

When an object is totally submerged in a fluid of density ρ_{fluid}, the volume V_{disp} of the displaced fluid is equal to the volume V_{obj} of the object; so, from Equation 3.4, the magnitude of the upward buoyant force is $B = \rho_{\text{fluid}}\, g\, V_{\text{disp}}$. If the object has a mass M and density ρ_{obj}, its weight is equal to $F_g = Mg = \rho_{\text{obj}}\, g V_{\text{obj}}$, and the net force on the object is $B - F_g = (\rho_{\text{fluid}} - \rho_{\text{obj}}) g V_{\text{obj}}$.

- Hence, if the density of the object is less than the density of the fluid ($\rho_{\text{obj}} < \rho_{\text{fluid}}$), the downward gravitational force is less than the buoyant force and the unsupported object accelerates upward as shown in the opposite Figure.



- If the density of the object is greater than the density of the fluid ($\rho_{obj} > \rho_{fluid}$), the upward buoyant force is less than the downward gravitational force and the unsupported object sinks as shown in the opposite Figure.



- If the density of the submerged object equals the density of the fluid, the net force on the object is zero and the object remains in equilibrium. Therefore, the direction of motion of an object submerged in a fluid is determined only by the densities of the object and the fluid. Thus, **the direction of motion of an object submerged in a fluid is determined only by the densities of the object and the fluid.**

Case 2: Floating Object

Now consider an object of volume V_{obj} and density $\rho_{obj} < \rho_{fluid}$ in static equilibrium floating on the surface of a fluid—that is, an object that is only partially submerged as shown in the opposite Figure. In this case, the upward buoyant force is balanced by the downward gravitational force acting on the object. If V_{disp} is the volume of the fluid displaced by the object (this volume is the same as the volume of that part of the object that is beneath the surface of the fluid), the buoyant force has a magnitude $B = \rho_{fluid}\, g V_{fluid}$. Because the weight of the object is $F_g = Mg = \rho_{obj}\, g V_{obj}$, and because $F_g = B$, we see that $\rho_{fluid}\, g\, V_{disp} = \rho_{obj}\, g V_{obj}$, or

$$\frac{V_{disp}}{V_{obj}} = \frac{\rho_{obj}}{\rho_{fluid}} \qquad (3.6)$$

This equation shows that **the fraction of the volume of a floating object that is below the fluid surface is equal to the ratio of the density of the object to that of the fluid**.

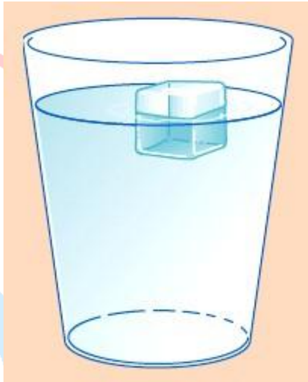
Quick Quiz 3.4

An apple is held completely submerged just below the surface of a container of water. The apple is then moved to a deeper point in the water. Compared to the force needed to hold the apple just below the surface, the force needed to hold it at a deeper point is (a) larger (b) the same (c) smaller (d) impossible to determine.

Quick Quiz 3.5

A glass of water contains a single floating ice cube. When the ice melts, does the water level (a) go up (b) go down

(c) remain the same?



3.7 Surface Tension

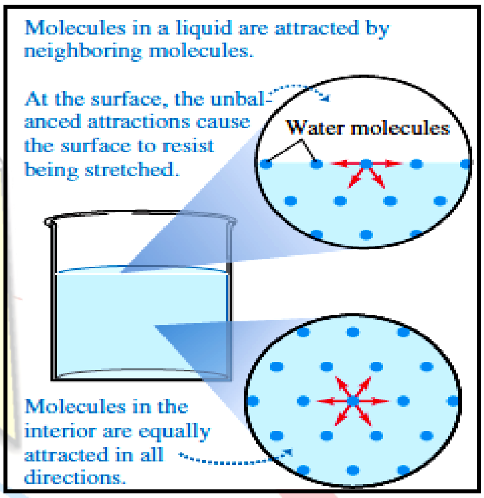
An object less dense than water, such as an air-filled beach ball, floats with part of its volume below the surface. Conversely, a paper clip can rest *atop* a water surface even though its density is several times that of water. This is an example of *surface tension*: The surface of the liquid behaves like a membrane under tension, and this tension, acting parallel to the surface. Surface tension arises because the molecules of the liquid exert attractive forces on each other. Because of surface tension, insects can walk on water; and objects more dense than water, such as a steel needle, can actually float on the surface.

More specifically, a quantity called the ***surface tension***, γ (the Greek letter gamma), is defined as the force F per unit length L that acts across any line in a surface, tending to pull the surface closed:

$$\gamma = \frac{F}{L} \tag{3.7}$$

Surface tension explains why freely falling raindrops are spherical (*not* teardrop-shaped): A sphere has a smaller surface area for its volume than any other shape. It also explains why hot, soapy water is used for washing. To wash clothing thoroughly, water must be forced through the tiny spaces between the fibers. To do so requires increasing the surface area of the water, which is difficult to achieve because of surface tension. The job is made easier by increasing the temperature of the water and adding soap, both of which decrease the surface tension.

We can see how surface tension arises by examining the process from the molecular point of view. The molecules of a liquid exert attractive forces on each other: these attractive forces are shown acting, in the opposite Figure, on a molecule deep within the liquid and on a second molecule at the surface. The molecule inside the liquid is in equilibrium due to the forces of other molecules acting in all directions. The molecule at the surface is also normally in equilibrium (the liquid is at rest). This is true even though the forces on a molecule at the surface can be exerted only by molecules below it (or along-



side it). Hence there is a net attractive force downward, which tends to compress the surface layer slightly-but only to the point where this downward force is balanced by an upward (repulsive) force due to close contact or collision with the molecules below. This compression of the surface means that, in essence, the liquid minimizes its surface area. This is why water tends to minimize its surface area and form spherical droplets, just as a stretched membrane does.

In order to increase the surface area of a liquid, a force is required and work must be done to bring molecules from the interior to the surface. This work increases the potential energy of the molecules and is sometimes called *surface energy*. The greater the surface area, the greater the surface energy. The amount of work needed to increase the surface area can be calculated by:

$$W = \gamma \Delta A$$

Where ΔA is the total increase in area. So we can write

$$\gamma = \frac{W}{\Delta A} \qquad \textbf{(3.8)}$$

Thus, the surface tension γ is not only equal to the force per unit length; it is also equal to the work done per unit increase in surface area. Hence, γ can be specified in N/m or J/m^2.

Problems

1. Find the mass and weight of the air at 20°C in a living room with a 4 m × 5 m floor and a ceiling 3 m high.

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2. The standard kilogram is a platinum-iridium cylinder 39 mm in height and 39 mm in diameter. **What** is the density of the material?

---

3. The mass of a solid cube is 856 g, and each edge has a length of 5.35 cm. **Determine** the density of the cube in basic SI units.

---

4. A 50-kg woman balances on one heel of a pair of high heeled shoes. If the heel is circular and has a radius of 0.5 cm, **what** pressure does she exert on the floor?

---

5. The four tires of an automobile are inflated to a gauge pressure of 200 kPa. Each tire has an area of 0.024 m^2 in contact with the ground. **Determine** the weight of the automobile.

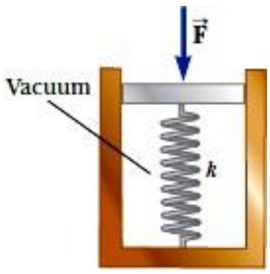
---

6. (a) **Calculate** the absolute pressure at an ocean depth of 1000 m. Assume the density of seawater is 1024 kg/m^3 and that the air above exerts a pressure of 101.3 kPa. (b) At this depth, **what** force must the frame around a circular submarine porthole having a diameter of 30 cm exert to counterbalance the force exerted by the water?

---

7. The surface tension γ of a liquid can be determined by measuring the force F needed to just lift a circular platinum ring of radius r from the surface of the liquid. (a) **Find** a formula for γ in terms of F and r. (b) at 30°C, if $F = 840 \times 10^{-3}$ N and r = 2.8 cm, **calculate** γ for the tested liquid.

8. The spring of the pressure gauge shown in the opposite Figure has a force constant of 1000 N/m, and the piston has a diameter of 2 cm. As the gauge is lowered into water, **what** change in depth causes the piston to move in by 0.5 cm?



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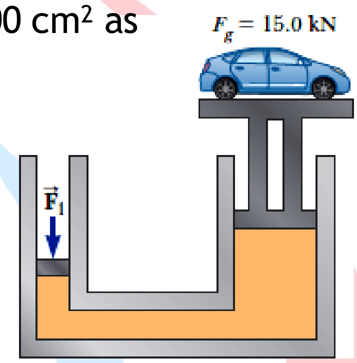
9. The small piston of a hydraulic lift has a cross-sectional area of 3 cm^2, and its large piston has a cross-sectional area of 200 cm^2 as shown in the opposite Figure. **What** force must be applied to the small piston for the lift to raise a load of 15 kN? (In service stations, this force is usually exerted by compressed air.)



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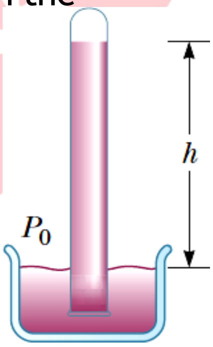
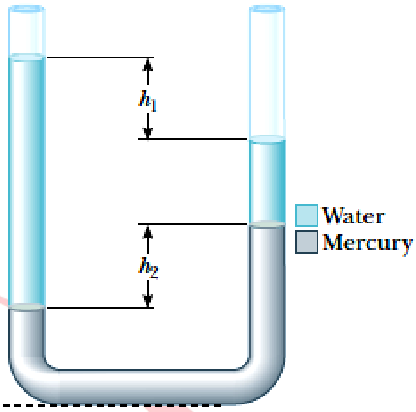
10. A very powerful vacuum cleaner has a hose 2.86 cm in diameter. With no nozzle on the hose, **what** is the weight of the heaviest brick that the cleaner can lift? (As shown in the opposite Figure)

---

11. Blaise Pascal duplicated Torricelli's barometer using a red Bordeaux wine, of density 984 kg/m^3, as the working liquid (As shown in the opposite Figure). (a) **What** was the height h of the wine column for normal atmospheric pressure? (b)Would you expect the vacuum above the column to be as good as for mercury?



12. A U-tube of uniform cross-sectional area, open to the atmosphere, is partially filled with mercury. Water is then poured into both arms. If the equilibrium configuration of the tube is as shown in the opposite Figure, with h_2 = 1cm, **determine** the value of h_1.



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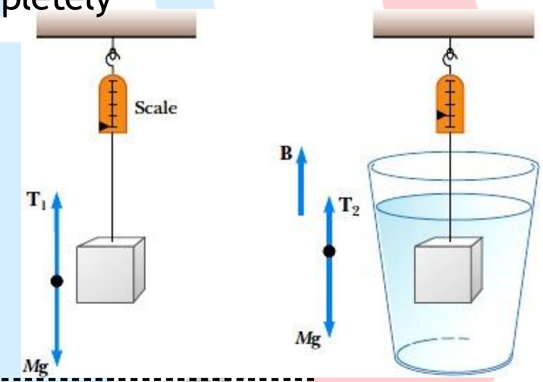
13. A Ping-Pong ball has a diameter of 3.8 cm and average density of 0.084 g/cm^3. **What** force is required to hold it completely submerged under water?

---

14. A piece of aluminum with mass 1 kg and density 2700 kg/m^3 is suspended from a string and then completely immersed in a container of water as shown in the opposite Figure. **Calculate** the tension in the string (a) before and (b) after the metal is immersed.



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15. A cube of wood having an edge dimension of 20 cm and a density of 650 kg/m^3 floats on water. (a) **What** is the distance from the horizontal top surface of the cube to the water level? (b) **How much** lead weight must be placed on top of the cube so that its top is just level with the water?

---

16. A plastic sphere floats in water with 50 % of its volume submerged. This same sphere floats in glycerin with 40 % of its volume submerged. **Determine** the densities of (a) the glycerin and (b) the sphere.

Chapter (4) Fluid Dynamics

Thus far, our study of fluids has been restricted to fluids at rest. We now turn our attention to fluids in motion.

4.1 Fluid Flow.

When fluid is in motion, its flow can be characterized as being one of two main types. The flow is said to be ***steady***, or ***laminar***, if each particle of the fluid follows a smooth path such that the paths of different particles never cross each other as shown in the opposite Figure. In steady flow, every fluid particle arriving at a given point in space has the same velocity.

Above a certain critical speed, fluid flow becomes ***turbulent***. Turbulent flow is irregular flow characterized by small whirlpool-like regions as shown in the opposite Figure. From this figure we observe that hot gases from a cigarette made visible by smoke particles. The smoke first moves in laminar flow at the bottom and then in turbulent flow above.

Application: Listening for Turbulent Flow

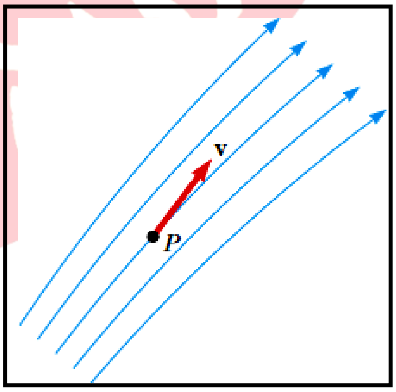
Normal blood flow in the human aorta is laminar, but a small disturbance such as a heart pathology can cause the flow to become turbulent. Turbulence makes noise, which is why listening to blood flow with a stethoscope is a useful diagnostic technique.

The term **viscosity** is commonly used in the description of fluid flow to characterize the degree of internal friction in the fluid. This internal friction, or viscous force, is associated with the resistance that two adjacent layers of fluid have to moving relative to each other. Viscosity causes part of the fluid's kinetic energy to be transformed to internal energy. This mechanism is similar to the one by which an object sliding on a rough horizontal surface loses kinetic energy. Whether a flow is laminar or turbulent depends in part on the fluid 's viscosity. The greater the viscosity, the greater the tendency for the fluid to flow in sheets or lamina and the more likely the flow is to be laminar.

Because the motion of real fluids is very complex and not fully understood, we make some simplifying assumptions in our approach. In our simplification model of **ideal fluid flow**, we make the following four assumptions:

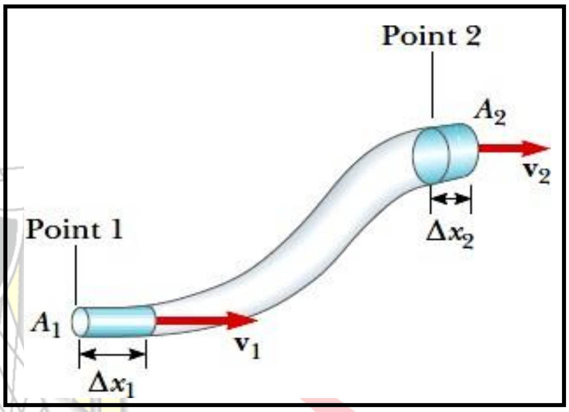
1. **The fluid is nonviscous.** In a nonviscous fluid, internal friction is neglected. An object moving through the fluid experiences no viscous force.
2. **The flow is steady.** In steady (laminar) flow, all particles passing through a point have the same velocity.
3. **The fluid is incompressible.** The density of an incompressible fluid is constant.
4. **The flow is irrotational.** In irrotational flow, the fluid has no angular momentum about any point. If a small paddle wheel placed anywhere in the fluid does not rotate about the wheel's center of mass, the flow is irrotational.

The path taken by a fluid particle under steady flow is called a **streamline**. The velocity of the particle is always tangent to the streamline as shown in the opposite Figure. A set of streamlines like the ones shown in this Figure form a *tube of flow*. Fluid particles cannot flow into or out of the sides of this tube; if they could, the streamlines would cross one another.



4.2 Equation of Continuity

Consider an ideal fluid flowing through a pipe of nonuniform size, as illustrated in the opposite Figure. The particles in the fluid move along streamlines in steady flow. In a time interval Δt, the fluid at the bottom end of the pipe moves a distance $\Delta x_1 = v_1 \Delta t$. If A_1 is the cross-sectional area in this region, then the mass of fluid contained in the left



blue region in this Figure is given by $m_1 = \rho A_1 \Delta x_1 = \rho A_1 v_1 \Delta t$, where ρ is the (unchanging) density of the ideal fluid. Similarly, the fluid that moves through the upper end of the pipe in the time interval Δt has a mass $m_2 = \rho A_2 \Delta x_2 = \rho A_2 v_2 \Delta t$. However, because the fluid is incompressible and because the flow is steady, the mass that crosses A_1 in a time interval Δt must equal the mass that crosses A_2 in the same time interval. That is, $m_1 = m_2$, or $\rho A_1 v_1 = \rho A_2 v_2$; this means that

$$A_1 v_1 = A_2 v_2 = \text{constant} \qquad (4.1)$$

This expression is called the **equation of continuity for fluids**. It states that

the product of the area and the fluid speed at all points along a pipe is constant for an incompressible fluid.

Equation 4.1 tells us that the speed is high where the tube is constricted (small A) and low where the tube is wide (large A). The product Av, which has the dimensions of volume per unit time, is called either the *volume flux* or the *flow rate*. The condition Av = constant is equivalent to the statement that the volume of fluid that enters one end of a tube in a given time interval equals the volume leaving the other end of the tube in the same time interval if no leaks are present.

You demonstrate the equation of continuity each time you water your garden with your thumb over the end of a garden hose as shown in the opposite Figure. By partially blocking the opening with your thumb, you reduce the cross-sectional area through which the water passes. As a result, the speed of the water increases as it exits the hose, and the water can be sprayed over a long distance.

Quick Quiz 4.1

You tape two different soda straws together end-to-end to make a longer straw with no leaks. The two straws have radii of 3 mm and 5 mm. You drink a soda through your combination straw. In which straw is the speed of the liquid the highest? (a) whichever one is nearest your mouth (b) the one of radius 3 mm

(c) the one of radius 5 mm (d) Neither—the speed is the same in both straws.

Quick Quiz 4.2

The volume per second of water that flows through each two pipes is the same. The flow velocity in the first pipe is one-quarter of that in the second pipe. What is the ratio of the radius of the first pipe to the radius of the second pipe?

(a) 2 (b) 4 (c) 1/2 (d) 1/4 (e) 1

Example 4.1 Niagara Falls

Each second, 5525 m^3 of water flows over the 670-m-wide cliff of the Horseshoe Falls portion of Niagara Falls. The water is approximately 2 m deep as it reaches the cliff. What is its speed at that instant?

Solution

The cross-sectional area of the water as it reaches the edge of the cliff is

$$A = (760\text{ m})(2\text{ m}) = 1340\text{ m}^2$$

The flow rate of water is 5525 m^3/s. This gives Since

$$\textbf{Flow rate} = Av$$

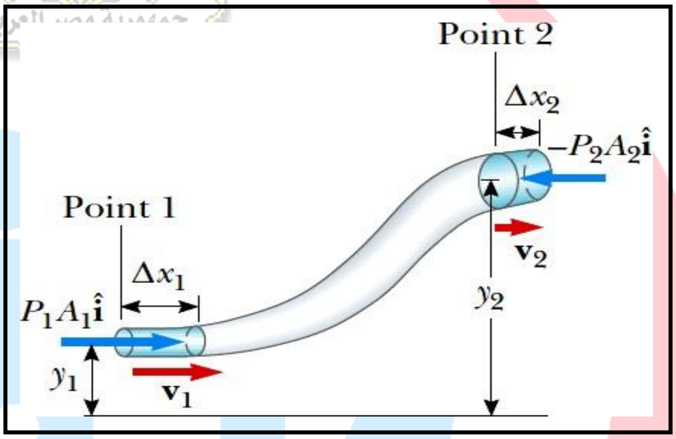
$$\therefore v = \frac{\text{Flow rate}}{A} = \frac{5525\ m^3/s}{1340\ m^2} \approx 4\ m/s$$

4.3 Bernoulli's Equation

You have probably experienced driving on a highway and having a large truck pass you at high speed. In this situation, you may have had the frightening feeling that your car was being pulled in toward the truck as it passed. We will investigate the origin of this effect in this section.

As a fluid moves through a region where its speed and/or elevation above the Earth's surface changes, the pressure in the fluid varies with these changes. The relationship between fluid speed, pressure, and elevation was first derived in 1738 by the Swiss physicist Daniel Bernoulli.

Consider the flow of a segment of an ideal fluid through a nonuniform pipe in a time interval Δt, as illustrated in the opposite Figure. At the beginning of the time interval, the segment of fluid consists of the blue shaded portion (portion 1) at the left and the unshaded portion. During the time interval, the left end of the segment moves to the right by a distance Δx_1, which is the length of the blue shaded portion at the left. Meanwhile, the right end of the segment moves to the right through a distance Δx_2, which is the length of the blue shaded portion (portion 2) at the upper right of this Figure. Thus, at the end of the time interval, the segment of fluid consists of the unshaded portion and the blue shaded portion at the upper right.



Now consider forces exerted on this segment by fluid to the left and the right of the segment. The force exerted by the fluid on the left end has a magnitude P_1A_1. The work done by this force on the segment in a time interval Δt is $W_1 = F_1\Delta x_1 = P_1A_1\Delta x_1 = P_1V$, where V is the volume of portion 1. In a similar manner, the work done by the fluid to the right of the segment in the same time interval Δt is $W_2 = - F_2\Delta x_2 = - P_2A_2\Delta x_2 = - P_2V$. (The volume of portion 1 equals the volume of portion 2.) This work is negative because the force on the segment of fluid is to the left and the displacement is to the right. Thus, the net work done on the segment by these forces in the time interval Δt is

$$W = (P_1 - P_2)V$$

Part of this work goes into changing the kinetic energy of the segment of fluid, and part goes into changing the gravitational potential energy of the segment– Earth system. Because we are assuming streamline flow, the kinetic energy of the unshaded portion of the segment in the above Figure is unchanged during the time interval. The only change is as follows: before the time interval we have portion 1 traveling at v_1, whereas after the time interval, we have portion 2 traveling at v_2. Thus, the change in the kinetic energy of the segment of fluid is

$$\Delta K = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$$

where m is the mass of both portion 1 and portion 2. (Because the volumes of both portions are the same, they also have the same mass.)

Considering the gravitational potential energy of the segment-Earth system, once again there is no change during the time interval for the unshaded portion of the fluid. The net change is that the mass of the fluid in portion 1 has effectively been moved to the location of portion 2. Consequently, the change in gravitational potential energy is

$$\Delta U = mgy_2 - mgy_1$$

The total work done on the system by the fluid outside the segment is equal to the change in mechanical energy of the system: $\boldsymbol{W = \Delta K + \Delta U}$. Substituting for each of these terms, we obtain

$$(P_1 - P_2)V = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 + mgy_2 - mgy_1$$

If we divide each term by the portion volume V and recall that $\rho = m/V$, this expression reduces to

$$P_1 - P_2 = \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2 + \rho g y_2 - \rho g y_1$$

Rearranging terms, we obtain

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2 \qquad (4.2)$$

This is **Bernoulli's equation** as applied to an ideal fluid. It is often expressed as

$$P + \frac{1}{2}\rho v^2 + \rho g y = \text{constant} \qquad (4.3)$$

Bernoulli's equation shows that the pressure of a fluid decreases as the speed of the fluid increases. In addition, the pressure decreases as the elevation increases. This latter point explains why water pressure from faucets on the upper floors of a tall building is weak unless measures are taken to provide higher pressure for these upper floors.

When the fluid is at rest, $v_1 = v_2 = 0$ and Equation 4.2 becomes

$$P_1 - P_2 = \rho g(y_2 - y_1) = \rho g h$$

This result is in agreement with Equation 3.3.

Although Equation 4.3 was derived for an incompressible fluid, the general behavior of pressure with speed is true even for gases: as the speed increases, the pressure decreases. This *Bernoulli effect* explains the experience with the truck on the highway at the opening of this section. As air passes between you and the truck, it must pass through a relatively narrow channel. According to the continuity equation, the speed of the air is higher. According to the Bernoulli effect, this higher speed air exerts less pressure on your car than the slower- moving air on the other side of your car. Therefore, there is a net force pushing you toward the truck!

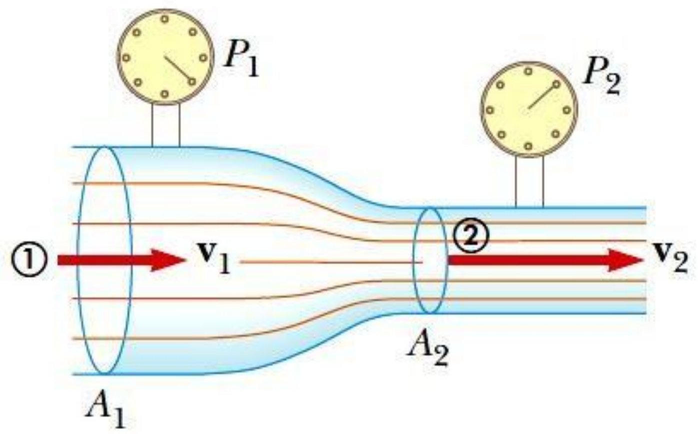
Quick Quiz 4.3

You observe two helium balloons floating next to each other at the ends of strings secured to a table. The facing surfaces of the balloons are separated by 1–2 cm. You blow through the small space between the balloons. What happens to the balloons? (a) They move toward each other. (b) They move away from each other. (c) They are unaffected.

4.4 Some Applications of Bernoulli's Equation Example 4.2

The Venturi Tube

The horizontal constricted pipe illustrated in the following Figure, known as a ***Venturi tube***, can be used to measure the flow speed of an incompressible fluid. **Determine** the flow speed at point 2 if the pressure difference P_1 - P_2 is known.



Solution

Bernoulli's equation shows how the pressure of an ideal fluid decreases as its speed increases. Therefore, we should be able to calibrate a device to give us the fluid speed if we can measure pressure.

Because the problem states that the fluid is incompressible, we can categorize it as one in which we can use the equation of continuity for fluids and Bernoulli's equation.

Apply Equation 4.2 to points 1 and 2, noting that $y_1 = y_2$ because the pipe is horizontal:

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2 \quad (1)$$

Solve the equation of continuity for v_1:

$$v_1 = \frac{A_2}{A_1} v_2 \quad (2)$$

Substitute this expression into Equation (1):

$$P_1 + \frac{1}{2}\rho\left(\frac{A_2}{A_1}\right)^2 v_2^2 = P_2 + \frac{1}{2}\rho v_2^2$$

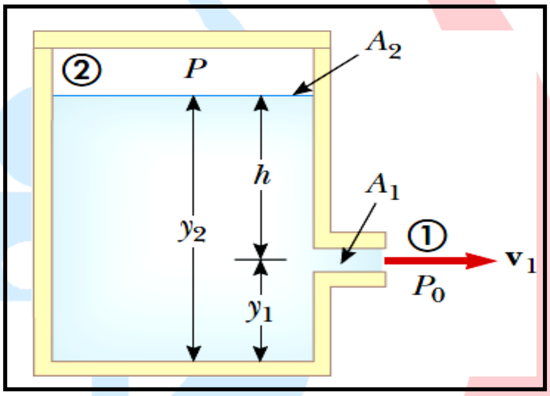
Solve for v_2:

$$v_2 = A_1 \sqrt{\frac{2(P_1 - P_2)}{\rho(A_1^2 - A_2^2)}}$$

From the design of the tube (areas A_1 and A_2) and measurements of the pressure difference P_1 - P_2, we can calculate the speed of the fluid with this equation. Because $A_2 < A_1$, Equation (2) shows us that $v_2 > v_1$. This result, together with Equation (1), indicates that $P_1 > P_2$. In other words, the pressure is reduced in the constricted part of the pipe.

Example 4.3 Torricelli's Law

An enclosed tank containing a liquid of density ρ has a hole in its side at a distance y_1 from the tank's bottom as shown in the opposite Figure. The hole is open to the atmosphere, and its diameter is much smaller than the diameter of the tank. The air above the liquid is maintained at a pressure P. **Determine** the speed of the liquid as it leaves the hole when the liquid's level is a distance h above the hole.



Solution

Imagine that the tank is a fire extinguisher. When the hole is opened, liquid leaves the hole with a certain speed. If the pressure P at the top of the liquid is increased, the liquid leaves with a higher speed. If the pressure P falls too low, the liquid leaves with a low speed and the extinguisher must be replaced.

Looking at the above Figure, we know the pressure at two points and the velocity at one of those points. We wish to find the velocity at the second point. Therefore, we can categorize this example as one in which we can apply Bernoulli's equation.

Because $A_2 >> A_1$, the liquid is approximately at rest at the top of the tank, where the pressure is P. At the hole, P_1 is equal to atmospheric pressure P_0.

Apply Bernoulli's equation between points 1 and 2:

$$P_0 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = P + \rho g y_2$$

Solve for v_1, noting that $y_2 - y_1 = h$:

$$v_1 = \sqrt{\frac{2(P - P_0)}{\rho} + 2gh}$$

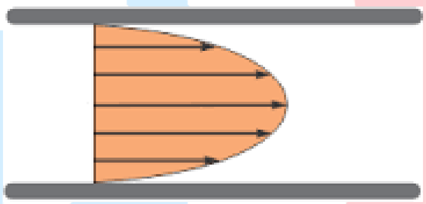
- When P is much greater than P_0 (so that the term $2gh$ can be neglected), the exit speed of the water is mainly a function of P.
- If the tank is open to the atmosphere, then $P = P_0$ and $v_1 = \sqrt{2gh}$. In other words, for an open tank, the speed of the liquid leaving a hole a distance h below the surface is equal to that acquired by an object falling freely through a vertical

  distance h. This phenomenon is known as **Torricelli's law**.

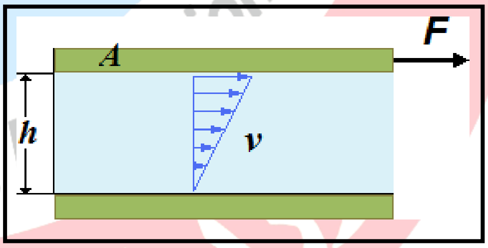
4.5 Viscosity

If you ever drifted in a boat on a gentle river, you may have noticed that your boat moved faster in the middle of the river than very close to the banks. Why would this happen? If the water in the river were an ideal fluid in laminar motion, it should make no difference how far away from shore you are. However, water is not quite an ideal fluid. Instead, it has some degree of "stickiness," called **viscosity**. For water, the viscosity is quite low; for heavy motor oil, it is significantly higher, and it is even higher yet for substances like honey, which flow very slowly. Viscosity causes the fluid streamlines at the surface of a river to partially stick to the boundary and neighboring streamlines to partially stick to one another. Therefore, the **viscosity** of a fluid is a measure of the fluid's resistance to flow or it is internal friction in a fluid. Viscous forces oppose the motion of one portion of a fluid relative to another. Viscous effects are important in the flow of fluids in pipes, the flow of blood, the lubrication of engine parts, and many other situations.

The velocity profile for the streamlines in viscous flow in a tube is sketched in the opposite Figure. The profile is parabolic, with the velocity approaching zero at the walls and reaching its maximum value in the center. This flow is still laminar, with the streamlines all flowing parallel to one another.



How is the viscosity of a fluid measured? The standard procedure is to use two parallel plates of area *A* and fill the gap of width *h* between them with the fluid. Then one of the plates is dragged across the other and the force *F* that is required to do so is measured. The resulting velocity profile of the fluid flow is linear as shown in the opposite Figure. The viscosity of different



fluids can be expressed quantitatively by a *coefficient of viscosity* η (the Greek lowercase letter eta). Coefficient of viscosity is defined as the ratio of the force per unit area divided by the velocity difference between the top and bottom plates over the distance between the plates:

$$\eta = \frac{F/A}{\Delta v/h} = \frac{Fh}{A\Delta v} \qquad (4.4)$$

The unit of η represents pressure (force per unit area) multiplied by time, or pascal seconds (Pa.s). This unit is also called a *poiseuille* (Pl).

Care must be taken to avoid confusing this SI unit with the cgs unit poise (P),

It is important to realize that the viscosity of any fluid depends strongly on temperature. You can see an example of this temperature dependence in the kitchen. If you store olive oil in the refrigerator and then pour it from the bottle, you can see how slowly it flows. Heat the same olive oil in a pan, and it flows almost as readily as water. Temperature dependence is of great concern for motor oils, and the goal is to have a small temperature dependence. Lava is an example of a viscous fluid. The viscosity decreases with

increasing temperature: The hotter the lava, the more easily it can flow. Table 4.1 lists some typical coefficient of viscosity values for different fluids. All values are those at room temperature ($20^{o}C$) except that of blood, whose value is given for the physiologically relevant temperature of human body temperature ($37\ ^{o}C$). incidentally, the coefficient of viscosity of blood increases by about 20% during a human's lifetime, and the average value for men is slightly higher than that for women (4.7×10^{-3} Pa.s vs. 4.3×10^{-3} Pa.s).

Table 4.1 **Some Typical Values of Coefficient of Viscosity at Room Temperature.**

Material	Coefficient of Viscosity (Pa.s)
Air	1.8×10^{-5}
Alcohol (ethanol)	1.1×10^{-3}
Blood (at body temperature)	4.0×10^{-3}
Honey	10
Mercury	1.5×10^{-3}
Olive oil	0.08
Water	1.0×10^{-3}

The viscosity of water is used as a reference to calculate other fluids' viscosity and is considered to be 1. The capsule of diarthrodial joints is normally filled with a fluid of viscosity 10 called synovial fluid. This fluid helps to reduce friction and wear of articulating surfaces. Just for comparison, the viscosity of olive oil, for example, is 84.

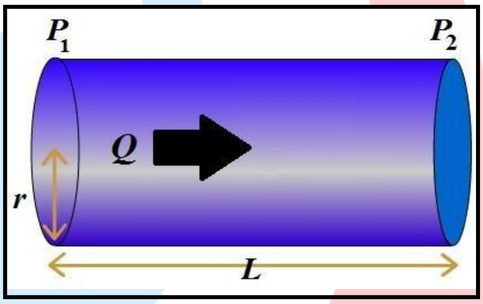
4.6 Poiseuille`s Equation

If a fluid had no viscosity, it could flow through a level tube or pipe without a force being applied. Because of viscosity, a pressure difference between the ends of a tube is necessary for the steady flow of any real fluid, be it water or oil in a pipe, or blood in the circulatory system of a human, even when the tube is level.

The rate of flow of a fluid in a round tube depends on the viscosity of the fluid, the pressure difference, and the dimensions of the tube. The French scientist J. L. Poiseuille (1799-1869), who was interested in the physics of blood circulation (and after whom the "poise" is named), determined how the variables affect the flow rate of an incompressible fluid undergoing laminar flow in a cylindrical tube. His result, known as *Poiseuille's equation*, is as follows:

$$Q = \frac{\pi r^4 (P_1 - P_2)}{8\eta L} \tag{4.5}$$

where r is the inside radius of the tube, L is its length, P_1-P_2 is the pressure difference between the ends, η is the coefficient of viscosity, and Q is the volume rate of flow (volume of fluid flowing past a given point per unit time). Equation 4.5 applies to laminar flow. There is no such simple mathematical relation if the flow is turbulent.



Poiseuille's equation tells us that the flow rate Q is directly proportional to the "*pressure gradient*," $(P_1$-$P_2)/L$, and it is inversely proportional to the viscosity of the fluid. This is just what we might expect. It may be surprising, however, that Q also depends on the *fourth* power of the tube's radius. This means that for the same pressure gradient, if the tube radius is halved, the flow rate is decreased by a factor of 16! Thus the rate of flow, or alternately the required to maintain a given flow rate, is greatly affected by only a small change in tube radius.

Blood Flow and Heart Disease

An interesting example of this r^4 dependence is ***blood flow*** in the human body. Poiseuille's equation is valid only for the streamline flow of an incompressible fluid with constant viscosity η; so it cannot be precisely accurate for blood whose flow is not without turbulence and that contains corpuscles (whose diameter is almost equal to that of a capillary). Hence η depends to a certain extent on the blood flow speed v. Nonetheless, Poiseuille's equation does give a reasonable first approximation. The body controls the flow of blood by means of tiny bands of muscle surrounding the arteries. Contraction of these muscles reduces the diameter of an artery and, because of the r^4 in Equation 4.5, the flow rate is greatly reduced for only a small change in radius. Very small actions by these muscles can thus control precisely the flow of blood to different parts of the body. Another aspect is that the radius of arteries is reduced as a result of arteriosclerosis (hardening of the arteries) and by cholesterol buildup; when this happens, the pressure gradient must be increased to maintain the same flow rate. If the radius is reduced by half, the heart would have to increase the pressure by a factor of about 16 in order to maintain the same blood-flow rate. The heat must work much harder under these conditions, but usually cannot maintain the original flow rate. Thus, high blood pressure is an indication both that the heart is working harder and that the blood-flow rate is reduced.

Example 4.4

Engine oil (its coefficient of viscosity is 0.2 Pa.s) passes through a fine 1.8 mm- diameter tube in a prototype engine. The tube is 5.5 cm long. What pressure difference is needed to maintain a flow rate of 5.6 mL/min?

Solution

The flow rate in SI units is $Q = 5.6\times10^{-6}\ m^3/60\ s = 9.33\times10^{-8}\ m^3/s$. We solve for P_1-P_2 in Eq. 4.5 and put all terms in SI units:

$$P_1 - P_2 = \frac{8\eta LQ}{\pi r^4}$$

$$= \frac{8(0.2\ \mathrm{N.s/m^2})(5.5 \times 10^{-2}\mathrm{m})(9.33 \times 10^{-8}\mathrm{m^3/s})}{3.14(0.9 \times 10^{-3}\mathrm{m})^4}$$

$$\approx 4 \times 10^3\ \mathrm{N/m^2}$$

or about 0.04 atm.

4.7 Reynolds number

If the flow velocity is large, the flow through a tube will become turbulent and Poiseuille's equation will no longer hold. When the flow is turbulent, the flow rate Q for a given pressure difference will be less than for laminar flow as given in Equation 4.5 because friction forces are much greater when turbulence is present.

The onset of turbulence is often abrupt and can be characterized approximately by the so-called ***Reynolds number***, *Re*:

$$\boldsymbol{Re = \frac{2\bar{v}r\rho}{\eta}} \quad \textbf{(4.6)}$$

Where $\bar{v}$ is the average speed of the fluid, ρ is its density, η is its coefficient of viscosity, and r is the radius of the tube in which the fluid is flowing. Notice that

Reynolds number *Re* is a *dimensionless* quantity. This means that *Re* has no units.

Experiments show that the flow is laminar if *Re* has a value less than about 2000, but is turbulent if *Re* exceeds this value.

Example 4.5

The average speed of the blood in the aorta (r = 1 cm) during the resting part of the heart's cycle is about 30 cm/s. Is the flow laminar or turbulent?. Density of the blood is 1.05×10^3 kg/m^3.

Solution

To answer this, we calculate the Reynolds number using Equation 4.6:

$$\boldsymbol{Re} = \frac{2(30 \times 10^{-2}\ \mathrm{m/s})(1 \times 10^{-2}\mathrm{m})(1.05 \times 10^3\ \mathrm{kg/m^3})}{(4 \times 10^{-3}\ \mathrm{N.s/m^2})} = 1600$$

The flow will probably be laminar, but is close to turbulence.

Problems

1. A horizontal pipe 10 cm in diameter has a smooth reduction to a pipe 5 cm in diameter. If the pressure of the water in the larger pipe is 8×10^4 Pa and the pressure in the smaller pipe is 6×10^4 Pa, **at what speed** does water flow through the larger pipe?

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2. A large storage tank, open at the top and filled with water, develops a small hole in its side at a point 16 m below the water level. If the rate of flow from the leak is equal to 2.5×10^{-3} m^3/min, **determine** (a) the speed at which the water leaves the hole and (b) the diameter of the hole.

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3. Water flows through a fire hose of diameter 6.35 cm at a rate of 0.012 m^3/s. The fire hose ends in a nozzle of inner diameter 2.2 cm. **What** is the speed with which the water exits the nozzle?

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4. The radius of the aorta is about 1 cm and the blood flowing through it has a speed of about 30 cm/s. **Calculate** the average speed of the blood in the capillaries given that, although each capillary has a diameter of about 8×10^{-4} cm, there are literally billions of them so that their total cross section is about 2000 cm^2.

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5. The opposite Figure shows a stream of water in steady flow from a kitchen faucet. At the faucet the diameter of the stream is 0.96 cm. The stream fills a 125-cm^3 container in 16.3 s. **Find** the diameter of the stream 13 cm below the opening of the faucet.

6. Through a pipe 15 cm in diameter, water is pumped from the Colorado River up to Grand Canyon Village, located on the rim of the canyon. The river is at an elevation of 564 m, and the village is at an elevation of 2096 m. (a) **What** is the minimum pressure at which the water must be pumped if it is to arrive at the village? (b) If 4500 m^3 are pumped per day, **what** is the speed of the water in the pipe?

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7. A Venturi tube may be used as a fluid flow meter. If the difference in pressure is $P_1 - P_2 = 21$ kPa, **find** the fluid flow rate in cubic meters per second, given that the radius of the outlet tube is 1 cm, the radius of the inlet tube is 2 cm, and the fluid is gasoline ($\rho = 700$ kg/m^3).

---

8. According to the plate tectonic model, the plates supporting the earth's continents move very slowly on the hot deformable rock below. **Show** that this flow is laminar using the following data: speed v = 50 mm/yr, density and viscosity of deformable rock below are $\rho = 3200$ kg/m^3 and $\eta = 4\times10^{19}$ Pa.s, with thickness ≈ 100 km.

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9. **What** must be the pressure difference between the two ends of a 1.9-km section of pipe, 35 cm in diameter, if it is to transport oil ($\rho = 950$ kg/m^3,

$\eta = 0.2$ Pa.s) at a rate of 450 cm^3/s?

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